

# Interpretation of Fractal Dimensions in Poverty Estimation\*

<sup>1</sup>Baceledes Estal, <sup>1</sup>Ariston S. Ronquillo Jr. <sup>2</sup>Joel B. Serad, <sup>2</sup>Brian A. Vasquez and <sup>3</sup>Le Roy Gerald A. Sarchez

---

## **Abstract**

*The paper analyzed the poverty incidence data among families in the Philippines across the different provinces for three periods: 2006, 2009, and 2012, using a surrogate measure of data roughness through fractal dimensions. Fractal dimensions as statistical quantities are robust measures which are not easily influenced by extreme observations unlike other variance-based poverty measures. Results showed that poverty incidence is most pronounced and severe in 2009 and significantly least severe in 2012. This did not coincide with the classical analysis of the National Statistical Coordination Board which averred no significant improvement in the poverty situation in the Philippines using the same data report. Moreover, multifractal spectrum analysis revealed that some provinces which used to be members of the 20 poorest provinces were delisted in 2012. The attribution of the improvement in the poverty situation are discussed in the paper considering some of the poverty alleviation projects in the Arroyo administration and the current Aquino administration's poverty alleviation thrusts.*

*Keywords: poverty incidence, income-based poverty measure, fractal statistics*

---

## **1.0 Introduction**

Poverty estimation remains an active area of research in the social sciences. There are generally two (2) types of poverty estimates: income-based estimators and deprivation-based estimators (UNDP, 2010) and their correlation have been found to be high (Mirasol, 2012). The latter finding implies that a relatively high proportion of the variance in deprivation-based estimates can be explained by income-based estimates of poverty. From the standpoint of national and international monitoring of poverty, it is more convenient to focus on income rather than on multidimensional indices that require information on variables that are difficult to obtain on the national level. Further, the fact that income-based poverty estimates explain most of the variance in the other poverty measurements justify using "income" as basis for poverty estimation.

Specifically, we focus on poverty measurement based on income disparities. The poverty measure based on the income difference (range) of the "rich" and the "poor", essentially captures the income gap between those considered poor and those considered rich. The wider the gap is, the more severe is the poverty situation. This traditional measure of poverty has been criticized by many (see for example, De Dios, 1993) as ignoring the bulk of the "poor" and therefore, giving a poor picture of the poverty situation. To overcome this criticism, other authors used the variance (of the income) as basis for developing a poverty measure (Balisacan, 2007) which takes into account deviations from a national income average. However, poverty estimates based on the variance of income are prone to the effects of outliers and extreme values which distort the true poverty

---

<sup>1</sup>Surigao del Sur State University

<sup>2</sup>University of the Visayas

<sup>3</sup>University of San Jose Recoletos

\*With assistance and guidance from Dr. Roberto N. Padua, who communicated the problem to the researchers and hinted on the use of fractal statistics.

situation. In fact, this situation was one of the main reasons for developing poverty indices that are not easily distorted by extreme values.

Roughly stated, we wish to derive a poverty index (based on income) that is not grossly affected by extreme values and which take into account the bulk of the rich in relation to the bulk of the poor. In this paper, we examine the use of fractal statistics, specifically, fractal dimensions in describing the poverty situation in the country. The fractal dimension of an object is an index of roughness of the features of the object (Mandelbrot, 1967). Rougher objects that have high shape irregularities will have higher fractal dimensions. The usual Euclidean shapes ( lines, circles and squares) are smooth regular-shaped objects that will possess integer dimensions e.g. dim =1 for a line, dim =2 for squares and circles, and dim =3 for cubes. In contrast, irregular shaped geometric objects such as snowflakes, plant leaves, and cloud formations will have fractional dimensions e.g. dim = 1.626 for snowflakes.

The translation of these purely geometrical concepts to numerical information is made possible through fractal statistics (Padua, 2012). Given a set of observations, say poverty indices by province, the one dimensional plot of these observations will reveal the fragmentation of a straight line induced by the observations. The degree to which the line is fragmented by the observations is the fractal dimension of the statistical data. That is, the fractal dimension of the data set represents data roughness and is similar to the variance-based poverty measure but with the advantage of not being dependent on the existence of a mean (robustness against extreme observations).

## 2.0 Basic Concepts in Fractal Statistics

Fractal statistical analysis applies to situations where the mean or first moment does not exist. It also applies to situations where smaller fluctuations

dominate the larger ones . Padua (2012) suggested using a power law distribution similar to Pareto's distribution given by:

$$1....f(x) = \frac{\lambda-1}{\theta} \left(\frac{x}{\theta}\right)^{-\lambda}, \lambda > 0, \theta > 0, x \geq \theta$$

where  $\lambda$  is defined as the fractal dimension of  $X$  and  $\theta$  is the smallest (positive) value of the random variable.

The maximum likelihood estimator of  $\lambda$  is:

$$2.... \hat{\lambda} = 1 + \frac{1}{\log\left(\frac{x}{\theta}\right)}$$

so that each observation contributes to the fragmentation of the support  $X$ . Padua (2013) demonstrated that the distribution of the maximum likelihood estimators obey an exponential type of distribution so that both the mean and variance of the fractal dimensions exist.

A device called fractal spectrum or  $\lambda(s)$  spectrum was suggested by Padua et al.(2013) to identify locations on the support  $X$  where high data roughness or fragmentation occur and where smoothness appear to dominate. The spectrum is defined as:

$$3.... \lambda(s) = 1 - \frac{\log(1-\alpha)}{\log\left(\frac{x}{\theta}\right)} = 1 - \frac{\log(1-\alpha)}{s}$$

where  $X_\alpha$  is the  $\alpha$ th percentile of  $X$  and  $s = \log\left(\frac{x}{\theta}\right)$ .

Deviations from smoothness indicate the severity of poverty incidence in a given context. A test for deviation from smoothness i.e.  $H_0 : \lambda = 1$ , is suggested in the second paper of Padua (2012) and the reader is referred to the paper as provided in the list of references.

## 3.0 Research Design and Methods

In order to validate the proposed fractal poverty index (FPI) in this paper, we considered the

poverty indices provided for years 2006, 2009 and 2012 in the Philippines (Poverty Incidence Among Families by Province, NSCB-2013). The purpose is to investigate how fractal poverty index can help answer the question: Has the poverty situation in the Philippines changed over time or has it remained the same?

A one-dimensional plot of the poverty indices obtained in a given year by the National Statistics Office (NSO) is constructed. The plot is then used as an input to a fractal software FRAK.OUT available as a freeware from the NET. The output is a fractal dimension for the data set. Thus, three fractal dimensions are obtained:  $\lambda_{2006}$ ,  $\lambda_{2009}$  and  $\lambda_{2012}$ .

The deviation from smoothness:

$$d = (\lambda - 1) \times 100\%$$

is computed for each fractal dimension. The higher the percentage deviation is, the more severe the poverty situation becomes.

Finally, we attempt to locate the areas of high fractal dimensions (more pronounced poverty) and relatively smoother areas by computing for the fractal spectra of each data set.

#### 4.0 Results and Discussions

Figures 1 to 3 show the one-dimensional plots of the original poverty incidence among families by province for years 2006, 2009 and 2012. We note how the poverty incidence information induced roughness or fragmentation on the straight line interval for each of the years under consideration.

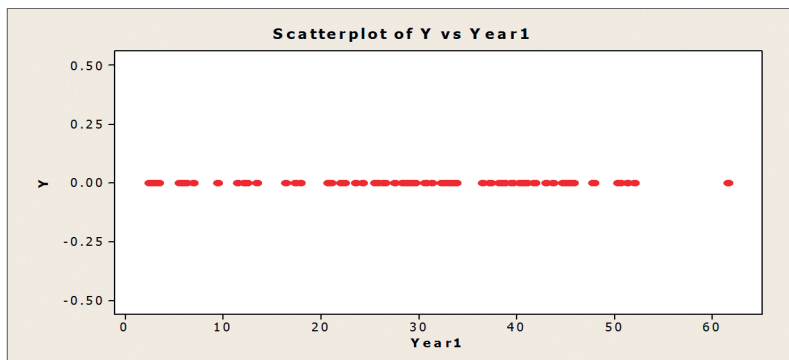


FIGURE 1: ONE-DIMENSIONAL PLOT OF POVERTY INCIDENCE BY PROVINCE FOR 2006

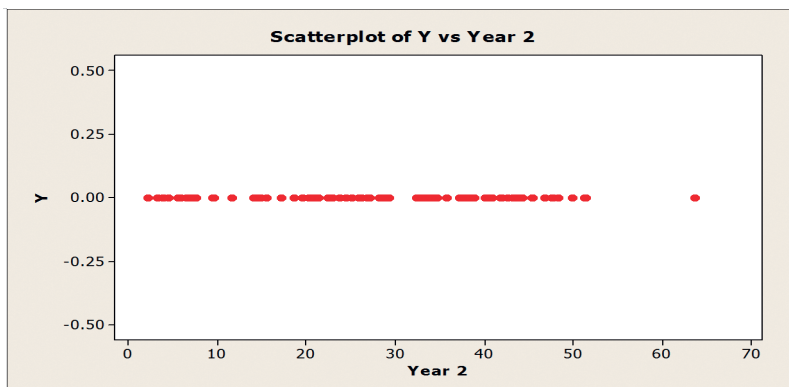


FIGURE 2: ONE-DIMENSIONAL PLOT OF POVERTY INCIDENCE BY PROVINCE FOR 2009

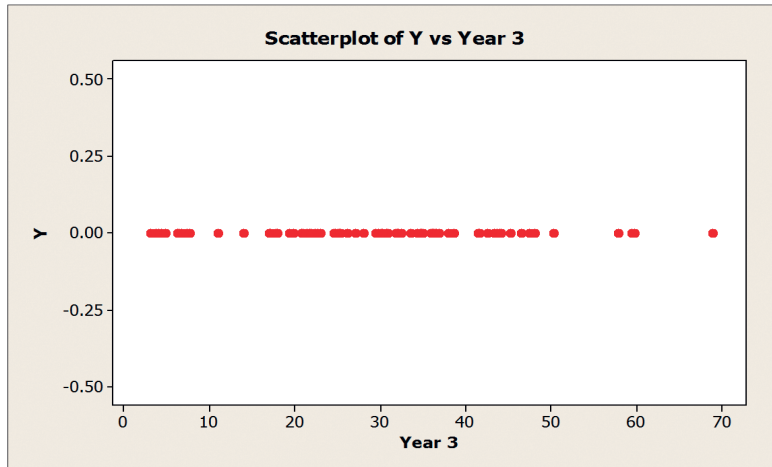


FIGURE 3: ONE-DIMENSIONAL PLOT OF POVERTY INCIDENCE BY PROVINCE FOR 2012

Table 1: Fractal Dimension and Deviation Statistics for the Poverty Incidence

YEAR	FRACTAL DIMENSION	DEVIATION STAT (%)
2006	1.4579	45.77%
2009	1.4679	46.79%
2012	1.4357	43.57%

Table 1, on the other hand, displays the fractal dimensions of the one-dimensional plots (induced roughness) as well as the deviation statistics (percentage departure from a smooth straight

line).

Figures 4 to 6 displays the fractal spectra for the poverty incidence for the years 2006, 2009 and 2012.

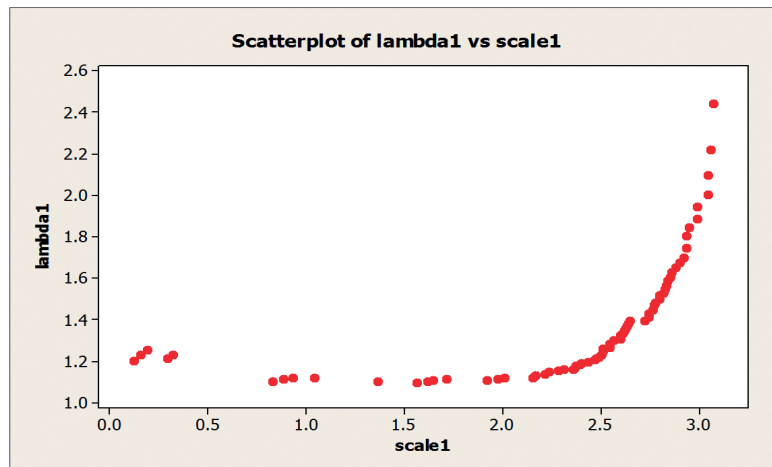


FIGURE 4: FRACTAL SPECTRUM FOR POVERTY INCIDENCE IN 2006

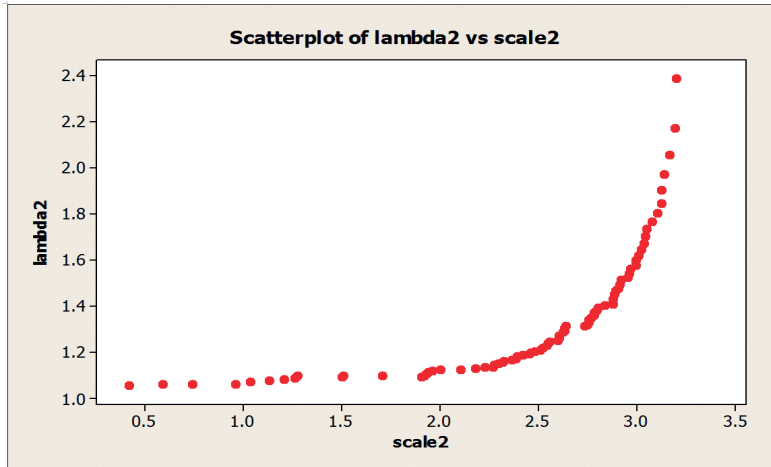


FIGURE 5: FRACTAL SPECTRUM FOR POVERTY INCIDENCE FOR 2009

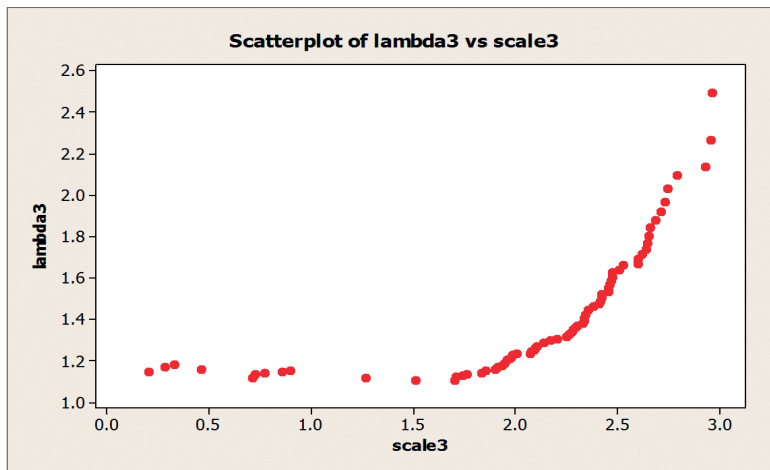


FIGURE 6: FRACTAL SPECTRUM OF POVERTY INCIDENCE FOR 2012

**5.0 Discussions**

1. The one-dimensional plots of the poverty incidence among families in the various provinces of the Philippines reflect a visual representation of the severity of the poverty situation in the country. Income disparities even among families whose incomes fall below a poverty threshold are most pronounced in 2009 (visually represented by the graphs as more fragmented points on a line) and least pronounced in 2012.
2. A quantitative index for comparing the degree of fragmentation ( and hence, of the degree of the severity of poverty) is the fractal dimension. Table 1 shows that the fractal dimensions of the poverty incidence increased from 1.4579 in 2006 to 1.4679 in 2009 or roughly, 1% increase in the severity of poverty experienced by the families in the provinces. The index dramatically decreased to 1.4357 in 2012 or roughly a 3.22% decrease in the

severity of poverty experienced by the poor in the country. "Severity" could be interpreted in various ways provided it is consistent with the geometric intent of measuring roughness. For instance, the drop in the fractal dimension between 2009 and 2012 ( 3% drop) could be interpreted as a 3% drop in the number of families having incomes below the poverty threshold in the provinces of the country. Testing the drop in the percentage of the number of poor families between 2009 and 2012 for significance yielded a t-value of  $t = 2.05641$  ( $p = .02$ ) significant beyond the .05 probability level.

3. Two (2) significant poverty alleviation programs, one each for the GMA and the PNOY administrations, were strongly pushed. In the period 2006 to 2009, the KALAHI program was espoused by the GMA administration. The Kapit-Bisig Laban sa Kahirapan (KALAHI) program is an anti- poverty initiative of the administration of President Gloria Macapagal-Arroyo. The project assisted the government in strengthening the capability of local government units (LGUs) to design, implement, and manage development activities. Such activities included basic infrastructure like new or rehabilitated roads and bridges, barangay schools and clinics, and water supply and sanitation facilities. Communities prioritized their development needs, design activities, seek technical expertise and make informed choices on how resources are used for sustainable poverty reduction. More than 5,300 poor barangays in 193 municipalities in 40 of the poorest provinces in the Philippines benefited from this project mostly in rural areas, where most poor Filipinos live. Because of the "empowerment" thrust of this project, benefits derived from the various sub-projects
- required long-term gestation periods i.e. benefits cannot be observed immediately but may be observable only after 5 to 10 years from project implementation. It is therefore possible that the drop in the severity of poverty incidence between 2009 and 2012 can be partly explained by the impact of the KALAHI project felt at the grassroots level.
4. According to the National Anti-Poverty Commission, in 2009, there were an estimated 66 programs that responded to the needs of the poor. This multiplicity of programs tends to result in poor coordination, as well overlapping and double counting of program beneficiaries, which reduces the effectiveness and efficiency of antipoverty programs over-all. During the Aquino administration, the NAPC systematized existing anti-poverty programs and built new programs, given existing gaps in anti-poverty interventions, especially in the areas of hunger and disaster relief and mitigation.
5. Realizing that while national economic growth rates have been significant even during the GMA administration, the NAPC concluded that it is in the area of redistribution of wealth that the Aquino administration must focus. For this reason, the Aquino administration launched its 4P program and other direct subsidy to the poor programs. Providing the poor with direct interventions across sectoral and geographical lines is, in terms of budget allocations, the most important component of the anti-poverty program. The programs under this strategy include Pantawid Pamilya, a conditional cash transfer program; subsidized PhilHealth coverage for indigents; a cash transfer program for poor senior citizens; and various hunger mitigation programs.

6. The drop in the fractal dimension of poverty in 2012 can be largely attributed to this Aquino-initiative of using direct subsidies to the poor.
7. Finally, the fractal spectrums revealed changes in the complexity and data roughness of the poverty incidence among families between 2009 and 2012. Variations in the poverty incidence particularly among the 20 poorest provinces identified in 2009 have been reduced significantly in 2012.

## 6.0 Conclusion

The use of fractal dimensions (and deviation statistics) for describing the poverty situation is practical with simple interpretation. Fractal dimensions have the advantage of being robust to the influence of extreme observations which often occur in income-based poverty estimation.

The Philippine poverty situation among families in the provinces have been significantly alleviated through the strategy of redistribution of wealth by the Aquino administration (CCT or 4P programs) as supported by the significant drop in the fractal dimension of poverty incidence between 2009 and 2012. This conclusion is quite different from the conclusions derived by the National Statistical Coordination Board. The difference in the conclusions is attributed to the difference in the analytic methods used by the present paper and the NSCB's classical approach.

## ACKNOWLEDGMENT

The authors are grateful to Dr. Roberto N. Padua who guided them in the intricacies of fractal statistical analysis.

## References

Ahmed, Akhter, Agnes Quisumbing, James Villafuerte and Rena de la Cruz-Dona (2004b). "Strengthening Social Protection in the Philippines," Policy note to

the World Bank in the Philippines, unpublished

Albert, Jose Ramon and Andre Philippe Ramos (2010). "Trends in Household Vulnerability," Philippine Institute for Development Studies, Discussion Paper Series No. 2010-01

Asian Development Bank (2006). Poverty in the Philippines: Incomes, Assets and Access. Manila: Asian Development Bank

Asian Development Bank (2009). Poverty in the Philippines: Causes, Constraints, Opportunities. Mandaluyong City: Asian Development Bank

Asian Development Bank (2010). Poverty in the Philippines: Causes, Constraints and Opportunities. Manila: Asian Development Bank

Balisacan, Arsenio (2007). "Why Does Poverty Persist in the Philippines? Facts, Fancies and Policies," Agricultural and Development Discussion Paper Series 2007-1, Southeast Asian Regional Center for Graduate Study and Research in Agriculture

Balisacan, Arsenio, Rosemarie Edillion, Alex Brilliantes and Dante Canlas (2002). Approaches to Targeting the Poor. Makati: National Economic and Development Authority and the United Nations Development Program

Balisacan, Arsenio and Nobuhiku Fuwa (2004). "Going Beyond Cross Country Averages: Growth, Inequality and Poverty Reduction in the Philippines," World Development 32 (11), pp. 1891-1907

Balisacan, Arsenio, Sharon Faye Piza, Dennis Mapa, Carlos Abad Santos and Donna Mae Odra (2010). "Tackling Poverty and Social Impacts: Philippine Response of the Crisis to the Global Economic Crisis," Report to the United Nations Development Program, Manila

Buendia, Emmanuel (2005). Democratizing Governance in the Philippines: Redefining and Measuring

the State of People's Participation in Governance.  
Quezon City: University of the Philippines

Canlas, Dante, Fernando Aldaba and Emmanuel Esguerra (2006). "Growth, Employment Creation and Poverty Reduction," Paper prepared for the National Employment Summit

Castro, Lina (2008). "2006 Poverty Statistics for the Basic Sectors," Presentation to the Users'Forum on the 2006 Poverty Statistics for the Basic Sectors and 2006 Child Development Index," Makati, Philippines.

De Dios, Emmanuel (1993). Poverty, Growth and the Fiscal Crisis. Makati: Philippine Institute for Development Studies

Di Gropello, Emanuela with Hong Tan and Prateek Tandon (2010). Skills for the Labor Market in the Philippines. Washington, D.C.: World Bank

Dy, Rolando (2011). "Philippine Agriculture and Fishery: Challenges, Opportunities and Global Competitiveness," Presentation to the Department of Agriculture.

Padua, Roberto, N. with Palompon, Daisy, R. and Ontoy, Dexter S. (2012) "Data Roughness and Fractal Statistics" (CNU Journal of Higher Education, Vol. 6, No.1, CHED-JAS Category A, pp. 87-101)

Padua, Roberto with Barabat, Efren (2013) " On the Properties of Multifractal Spectrum" (The Recoletos Journal of Higher Education, Vol. 1, No.1, Research Journal of the University of San Jose Recoletos, CHED-JAS Category B, pp. 78-89)

Similarity Index: 10%

Paper ID: 369133292