Fractal Volatility of Stock Price Levels

Kristine June D. Uy, Chris Rudyard F. Naval*

Abstract

Applying fractal statistics to the components of the Dow Jones Industrial Average that belong to the industries of telecommunications and technology, this study established the indicators of fractal distribution and examines the fractal spectrum of each of the stocks selected. From the said fractal spectrum, the researchers identified segments that represent similar fractal dimensions. The findings of this study gave an overall picture of the entire volatility level within selected stocks for the purpose of dissecting the said volatility level into smaller segments. Scrutinizing the fluctuations within a volatility level will be helpful for short term investors in identifying the price levels appropriate to their risk appetite. It is clearly manifested by the results of the fractal segmentation.

Keywords: Volatility, Fractals, Price Levels, Stocks, Stock Prices

JEL Classification: D4

1.0 Introduction

Volatility has been the subject of investor fascination all over the globe. Traders and speculators, even those not entirely well versed in the stock exchange have, at one point or another, been mesmerized by the dips and climbs that represent the downs and ups of the stock prices. To the imaginative, a graph of the historical stock prices may appear like the lines in a heart monitor, only more erratic, more irregular. In between each of these plunge and rise lie an investor's anxiety and anticipation, dread and relief. Indeed, volatility fuels the life of the stock market. Being the representation of risk, it embodies an opportunity to earn as well as an occasion to lose. This is perhaps the reason for the unending fascination with volatility and the overzealous anticipation of how the stock price will move next. No matter the investor attitude, all the players in the stock market want to have the ability to read through the signs that the volatility levels are sending in the hope that they would be able to capitalize on the profits that stock price changes bring, and to moderate the possible financial losses, if not all together avoid them, that come along with the movements in the said prices.

Volatility has been the theme of many research studies in the field of finance. It has been pointed out that, like most resources, changes in the stock prices are attributed to market forces. Finance has seen the advent of the efficient market hypothesis and behavioral finance, among others, in an attempt to explain stock price changes. The efficient market hypothesis posits that information plays a vital role in the pricing of stocks; information is deemed to be integrated into the stock prices. Behavioral finance, however, ventures into the vast sphere of psychology and looks into how share prices are driven by human factors such as emotions and prejudices. A study by Uy and Naval (2013) proposed to view stock price volatility in terms of the ruggedness of information as represented by its fractal dimension. It was determined that stocks from different industries exhibit different

*Faculty, Department of Accountancy and Finance, College of Commerce
University of San Jose Recoletos

fractal behaviors. The volatility levels of all stocks in the Dow Jones Industry Average belonging to the technology and telecommunications industries exhibited similar trends (Naval and Uy, 2013), suggesting that an industry's volatility levels mirror the stocks belonging to it. Volatility has also been seen to be affected by a number of factors. In Adjasi (2009), the increase and decrease in volatility as evidenced by the African markets is influenced by cocoa prices, interest rates, gold prices, oil prices, and money supply. A study of the effects on volatility on stock price limits conducted by Chang (2006) explored the Taiwan stock market to determine whether price limits result in the volatile spill over of stock prices.

True enough, finance has seen its fair share of studies that focus on the volatility of stock prices. It is to be noted that these studies ordinarily look at volatility in order to identify and understand the factors that contribute to the stock price movements, or attempt to measure volatility. This study, however, will aim to examine volatility levels within volatilities in stock prices. The researchers aim to give a snapshot of the entire volatility level within selected stocks with the purpose of dissecting the said volatility level into smaller segments. Scrutinizing the fluctuations within a volatility level will be helpful for short term investors in identifying the price levels appropriate to their risk appetite. The researchers propose to perform this by using fractal statistics.

2.0 Conceptual Framework

Roughness is the fabric in which the whole universe is stitched on. This fabric of roughness is woven in an interesting blend of seemingly random albeit self-similar patterns. A snapshot of the whole will show a beautiful ruggedness in the form of the world's coastlines, mountain formations, and rainforests, among others. A closer inspection, however, will show the same similar rugged beauty in the patterns of the waves, terrains, and leaves of trees. Interestingly, the whole tells the story of the parts and the parts tell the story of the whole. This scale invariant, self similar, and rugged roughness are what captured the interest of Benoit Mandelbrot (1967), a mathematician famous for his work on what we now know as fractals.

The stock market is an extension of this fractality that is so often associated with nature. A visual appreciation of a time series plot of historical stock prices will initially show an apparent jagged series of points that, to the untrained eye, may seem random and incoherent. Fractal geometry, however, will allow us to view the very same plot and see an apparently hidden pattern in the randomness of the high and low points embodying the stock prices and their changes. This pattern is present even when one would divide the plot into segments, showing that the repetition of these fluctuations are similar regardless of time interval. It is for this reason that the researchers find it appropriate to employ the use of fractal statistics as developed by Padua (2013). While traditional statistics resort to smoothing the data and disregarding randomness, fractal statistics takes into account the obvious fact that the data sets are not all the time smooth. The presence of outliers is almost always assured. It is the belief of the researchers that said outliers should not be disregarded as they form an integral part of the data set analyzed. Thus, fractal statistics is the more appropriate tool to use.

There are a myriad of factors that affect stock price changes. Information is just one of the many. As the efficient market hypothesis points out, the market efficiency can either be weak, semi-strong, or strong depending on the type of information involved. Information, then, is held as an essential determinant of stock prices. Consequently, stock price changes are affected by the type and volume of information consumed and utilized by the players in the stock market. On the other hand, the researchers also agree that feedback and word-of-mouth can strongly influence the determination of a stock price. The enthusiasm displayed by those who have had speculative gains attracts public attention (Shiller, 2003). The life of a particular piece of information and how it flows thrives on the social support it gets (Uy, Chua, and Naval, 2013) thus, for as long as the information keeps making its rounds, its influence on the stock price remains.

Owing to the variety of factors that affect it, stock price fluctuations remain to hold an intriguing spot in the financial universe. The researchers endeavour to take a closer look into this phenomenon of volatility by sharpening the focus on the fluctuations within the volatility of the stocks selected. As such, the researchers shall examine the volatilities within the volatility using fractal statistics.

3.0 Methodology

The researchers will obtain the daily historical prices of the Dow Jones Industrial Average's Technology and Telecommunications stocks with time-frames from January 2008 to December 2012. The logic behind the decision in choosing these industries is their high volatility due to the nature of the stocks. Since advancements in technology occur frequently, the nature of these stocks' volatility is caused by the uncertainties in demands for technology services and products to which these companies provide. As a result, demands for these stocks fluctuate radically. These data sets have also been tested for fractal volatility similarities per industry by Naval and Uy (2013).

Determining the presence of a fractal distribution by using statistical indicators is the initial procedure in fractal analysis. Ascertaining the absence of a normal distribution is done by having an ocular inspection of the histograms of the closing prices of the stocks. Moreover, the researchers validated the results by submitting the data set to an additional normality test -- Kolmogorov-Smirnov Goodness of Fit Test. Once a fractal distribution is determined, the raw data (i.e. the historical prices) is translated by means of the fractal statistics formula.

$$\lambda = 1 - \frac{log\,(1-\alpha)}{log\,(x)}$$

Where:

$$\alpha = \frac{Rank\ of\ the\ Data}{n}$$

$$Scale = \frac{1}{Log\ (x)}$$

Equation 1 Fractal Dimension Formula

By fitting a line starting from the point with the lowest scale, the researchers identified segment of scales that possess the same volatility levels. The last point that does not fit to the first line is the beginning of the next segment. The procedure is repeated until the point with the highest scale is reached and all the segments are established. The slope of the segment is determined by using the formula on Equation 2 (Naval & Uy, 2013).

$$Slope = \frac{d\lambda}{d\,scale} = \frac{\lambda_{n+1} - \lambda_n}{scale_{n+1} - scale_n}$$

Equation 2 Fractal Slope Formula

The implication of the fractal slope is the level of fluctuation: the nearer the slope to zero, the lesser volatility of the segment (Naval & Uy, 2013). Utilizing fractal statistics, the researchers will investigate the fluctuation levels (risk) of each scale segment. The scale segments are identified by the proponents as the price levels of the historical stock price with similar volatility levels. The researchers will prove that the volatility of price levels within the data set supports their hypothesis on the fractal slope. The validation of the fractal slope hypothesis is cross-referenced from the fractal spectrum to the stock's time series plot. The initial process in cross-referencing is deriving the elbow's value, which is the starting or ending point of a segment, from the inversion formula. (See Equation 3)

$$x = e^{\frac{1}{s}}$$

Equation 3 Inversion Formula

The identified elbow values are the threshold levels of each price level. The threshold levels are established to the stock price's graph by plotting a horizontal line. This methodology is called fractal segmentation. The analyses and conclusions based on the derived slope of the fractal spectrum will be supported by the graph embedded with the threshold lines.

4.0 Presentation , Analysis and Interpretation of Data

The researchers identified fractal distributions using statistical indicators: optical assessment of histograms and the Kolmogorov-Smirnov Goodness of Fit Test. The initial process of identifying a fractal distribution is ascertaining the absence of a normal distribution by the ocular examination of the histograms of the closing stock prices. To qualify as a normal distribution, the data set's frequency must show a bell-shaped curve. The histograms are presented in Figures 1 to 7.

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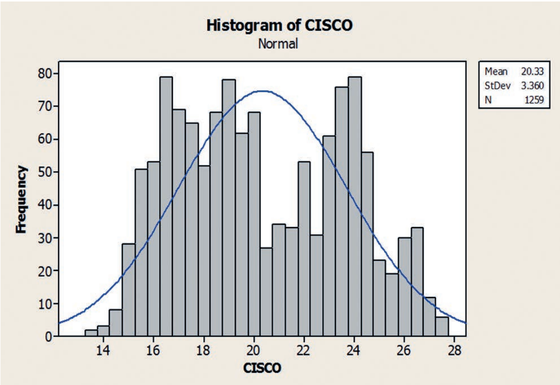

Figure 1 Histogram , CISCO

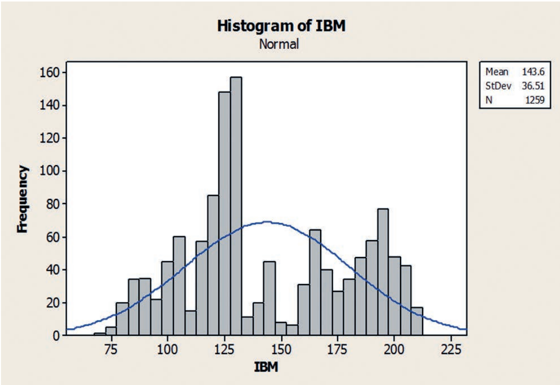

Figure 2 Histogram, IBM

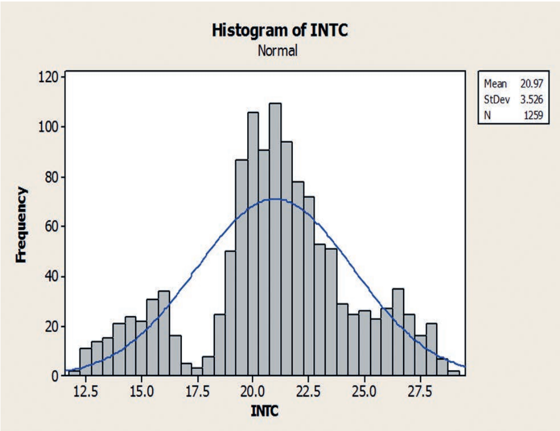

Figure 3 Histogram , Intel Corporation

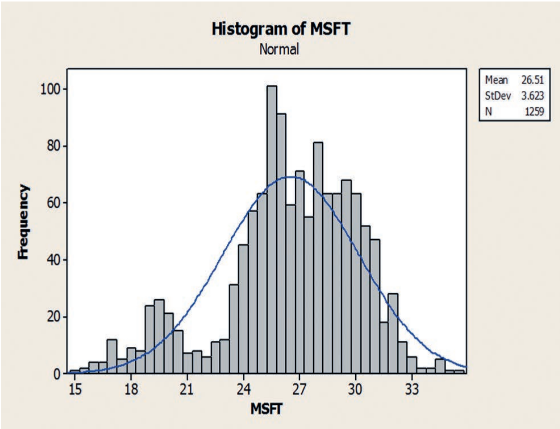

Figure 4 Histogram, Microsoft

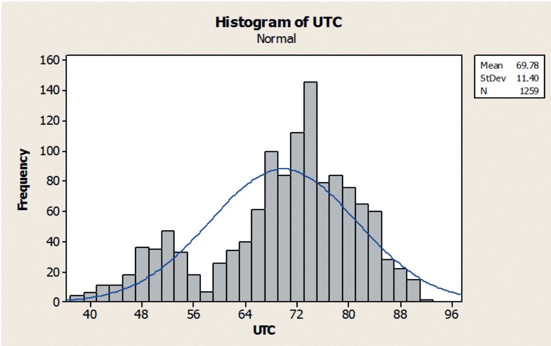

Figure 5 Histogram, United Technologies

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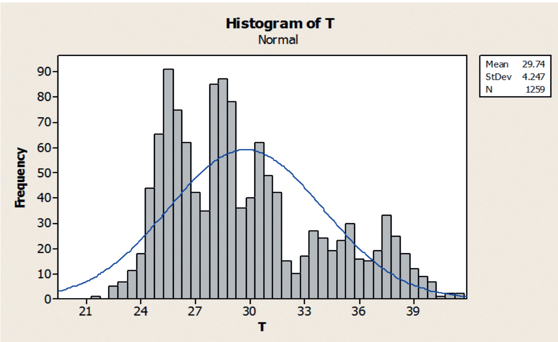

Figure 6 Histogram, AT & T

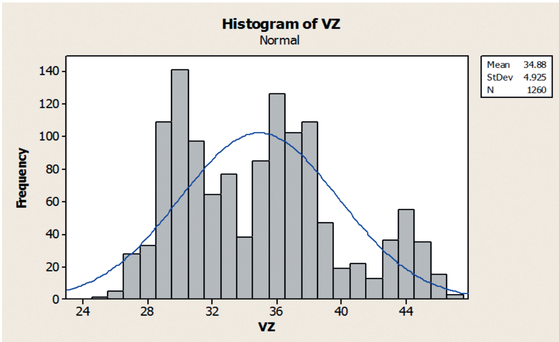

Figure 7 Histogram, Verizon

As seen on Figures 1 to 7, the conclusion of the data set's normality is rejected. This result might suggest the presence of a fractal distribution. Having this conclusion, the researchers will revalidate the non-normality of the data sets by performing another statistical examination -- the Kolmogorov-Smirnov Goodness of Fit Test. The fit tests are presented in Figures 8 to 14.

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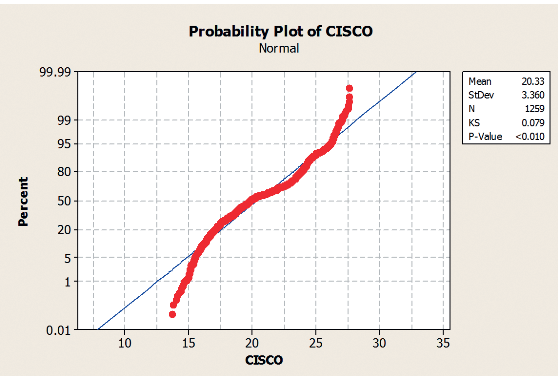

Figure 8 Normality Test, CISCO

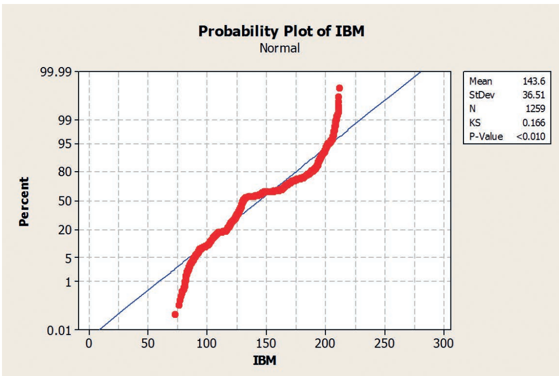

Figure 9 Normality Test , IBM

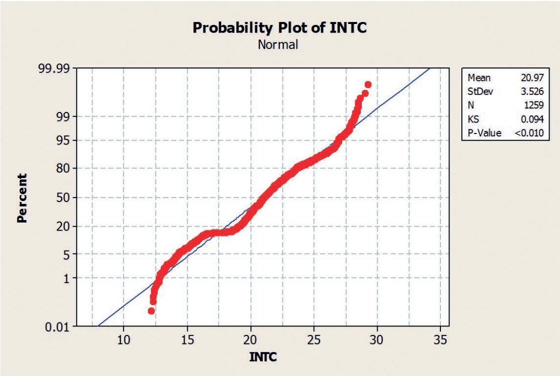

Figure 10 Normality Test, Intel Corporation

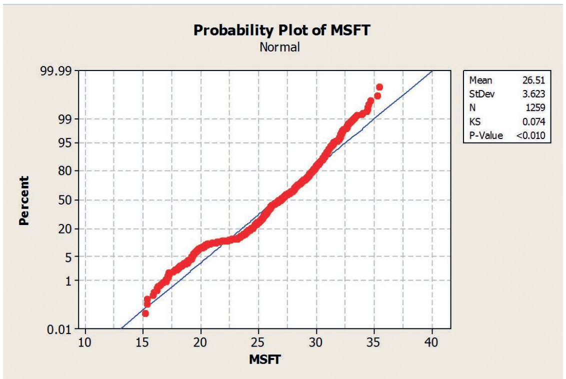

Figure 11 Normality Test, Microsoft

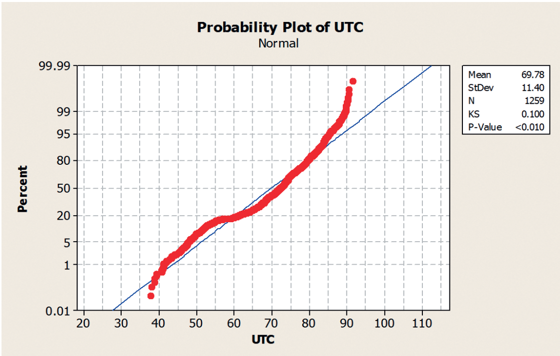

Figure 12 Normality Test, United Technologies

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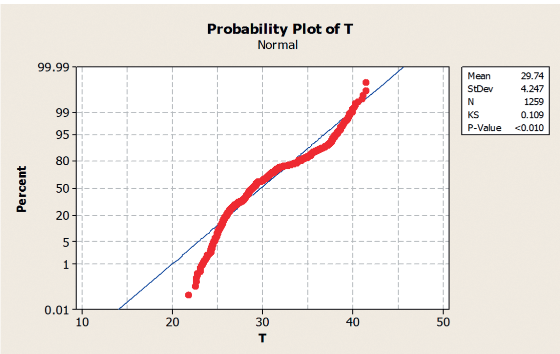

Figure 13 Normality Test, AT & T

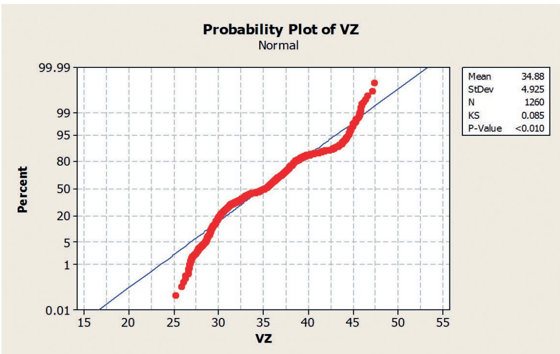

Figure 14 Normality Test, Verizon

As shown on Figures 8 to 14, it can be inferred that the data set's normality is rejected and verified that there exists a fractal distribution. The plots above manifest prominent distinctions from the data as compared to the theoretical distribution line (blue line). Moreover, the p values of less than 0.01 suggest non-normality of the data set. Supporting the inference of the researchers, the p values of less than 0.01 are exhibited by the observed stocks: CISCO, IBM, INTC, MSFT, UTC, T, and VZ. With the combined results of the histogram assessment and Kolmogorov-Smirnov Goodness of Fit Test, the researchers applied fractal statistics.

The raw data are translated using the Lambda formula to create each stock's fractal spectrum. The fitting of the line is also done to the generated fractal spectrum to identify the segment of the scales and its elbows. The identified segments are inferred to have the same volatility levels. The spectrum segmentation is presented on Figures 15 to 21.

The fractal behavior of the stocks is represented

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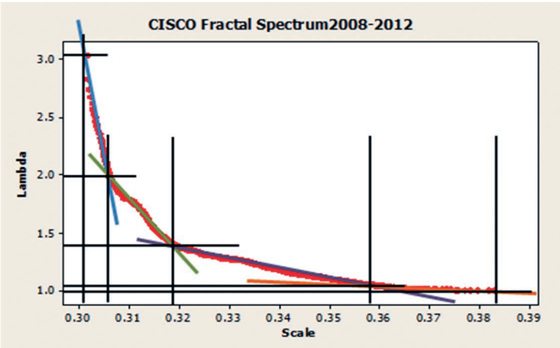

Figure 15 Fractal Spectrum, CISCO

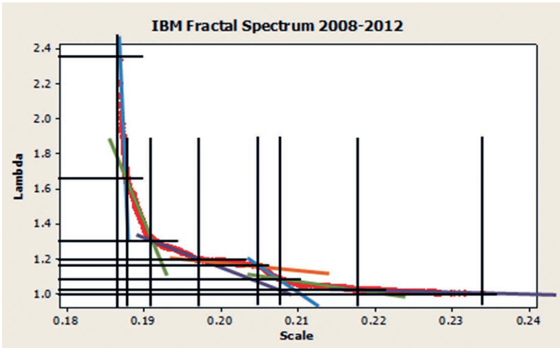

Figure 16 Fractal Spectrum, IBM

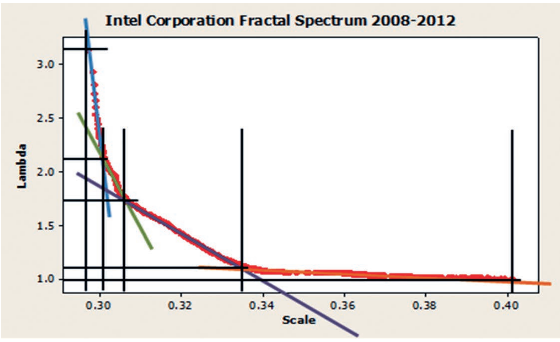

Figure 17 Fractal Spectrum, Intel Corporation

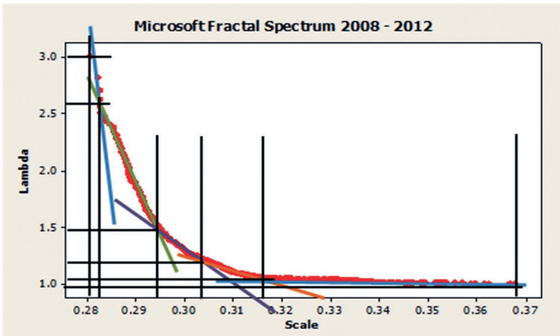

Figure 18 Fractal Spectrum, Microsoft

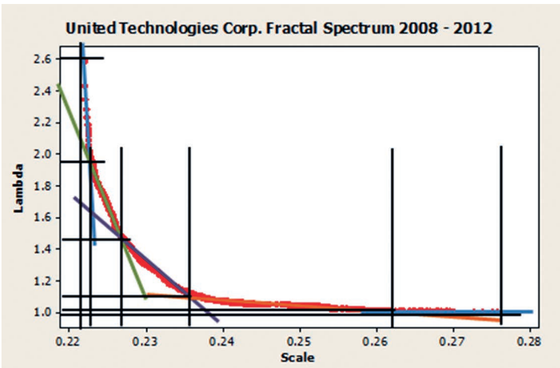

Figure 19 Fractal Spectrum, United Technologies

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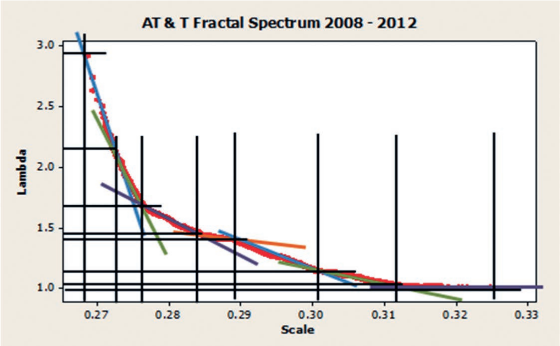

Figure 20 Fractal Spectrum, AT & T

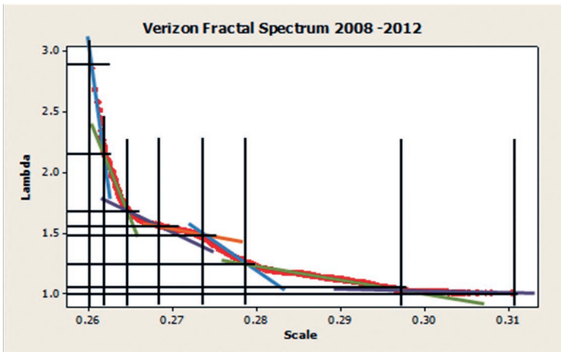

Figure 21 Fractal Spectrum, Verizon

by its fractal spectrum (Uy and Naval, 2013). Since this behavior gives a glimpse into the volatility of stocks through the lens of fractal dimension, the researchers categorized the spectrum into distinct movements within the overall behavior of the stocks. These categories are shown in Figures 15 to 21. The areas in between two vertical lines represent the segments identified in the fractal

Table 1 Elbows of each spectrum

	Elbows of each Spectrum							
	1	2	3	4	5	6	7	8
Cisco	0.301	0.306	0.319	0.3583	0.3835			
IBM	0.1868	0.188	0.191	0.1973	0.205	0.2079	0.218	0.234
Intel	0.2985	0.3005	0.303	0.3175	0.4008			
Microsoft	0.2805	0.2825	0.2945	0.3035	0.3165	0.3685		
United Technology	0.2214	0.2228	0.227	0.2357	0.2621	0.2762		
AT & T	0.2685	0.2727	0.2763	0.284	0.2894	0.301	0.3117	0.3255
Verizon	0.26	0.2619	0.2649	0.2686	0.2737	0.279	0.2973	0.317

Table 2 Slopes of the Scale Segment

SLOPES							
Stocks	Scale Segment						
	1	2	3	4	5	6	7
Technology							
Cisco	-215	-45.77	-8.40	-2.78			
IBM	-575	-120	-17.46	-3.90	-27.59	-5.94	-1.88
Intel	-500	-150	-43.10	-1.50			
Microsoft	-212.5	-91.67	-30.56	-11.54	-1.54		
United Technology	-457.14	-116.67	-41.38	-3.41	-2.84		
Telecommunications							
AT & T	-184.52	-131.94	-29.22	-9.26	-23.71	-9.35	-3.62
Verizon	-381.58	-158.33	-33.78	-17.16	-42.45	-10.25	-3.81

spectrum, while the horizontal lines are used to point out the elbows of each segment. The elbows, which indicate the segment's starting point and end point, are identified to establish the scale segments. On the fractal spectrum, the first elbow is the point in the lowest scale, and the last elbow is the one in the highest scale. Together, the identification of the elbows and the segments allowed to researchers to examine the volatility of the stocks through a more microscopic view. For example, Figure 15 shows the fractal spectrum of CISCO. A visual inspection of the entire fractal spectrum gives the overall behavior of the stock; however, one can see shifts in the fractal behavior within the spectrum as evidenced by the gradual and irregular decays. Each major shift is characterized by its starting and end points (i.e. the elbows); the area in between the elbow is the segment. For CISCO, five segments are identified. Presented on Table 1 is the scale value of the elbows.

A tabular view of the segmentation performed in Figures 15 to 21 is shown in Table 1. The researchers extrapolated the values in the scale which represented the elbows in the segment by using the inversion formula in Equation 3. Segment 1 of IBM, then, is the area between scale value 0.1868 and 0.188; segment 2 of IBM starts with scale value 0.188 and ends with 0.191, and so on. With the information provided by Table 1, we can now compute for the slope of the scale segments. See Table 2.

Applying the hypothesis of the fractal slope, we can produce insights on the price levels' variability for each stock. For Cisco, the trend of

Table 3 Original Value of the Spectrum's Elbows

	Original Value of the Spectrum's Elbows							
	1	2	3	4	5	6	7	8
Cisco	27.72	26.26	22.98	16.30	13.57			
IBM	211.31	204.21	187.84	158.92	131.37	122.73	98.21	71.77
Intel	28.51	27.88	27.12	23.33	12.12			
Microsoft	35.34	34.46	29.83	26.97	23.56	15.09		
United Technology	91.53	88.97	81.88	69.59	45.39	37.36		
AT & T	41.45	39.14	37.31	33.82	31.67	27.72	24.73	21.59
Verizon	46.81	45.52	43.60	41.39	38.61	36.03	28.89	23.44

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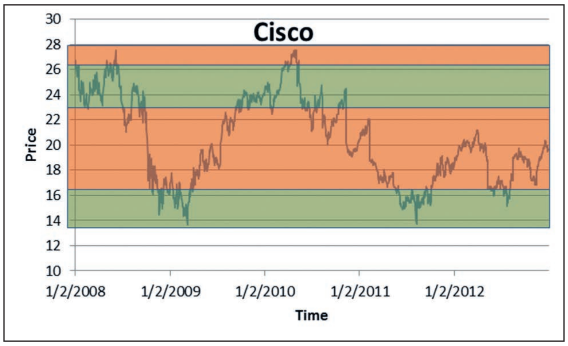

Figure 22 Fractal Segmentation, Cisco

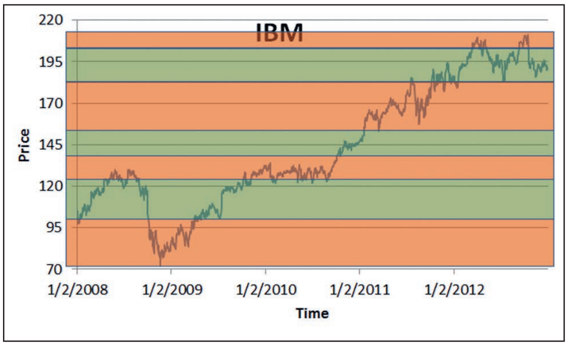

Figure 23 Fractal Segmentation, IBM

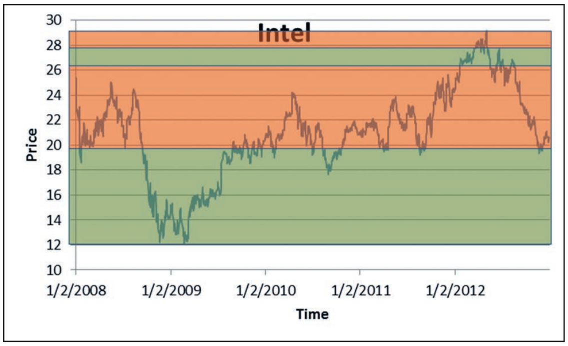

Figure 24 Fractal Segmentation, Intel

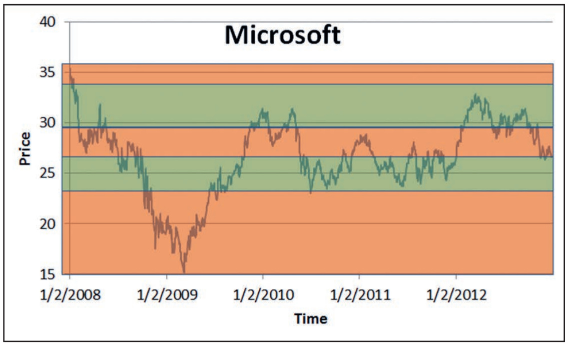

Figure 25 Fractal Segmentation, Microsoft

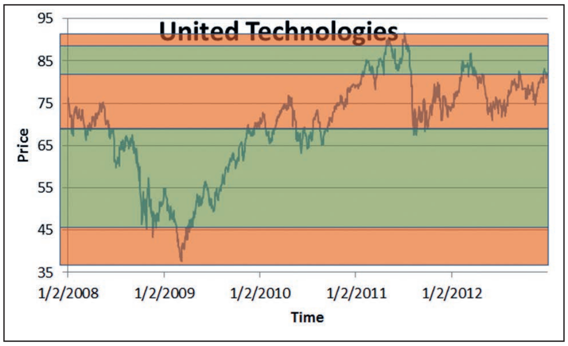

Figure 26 Fractal Segmentation, United Technologies

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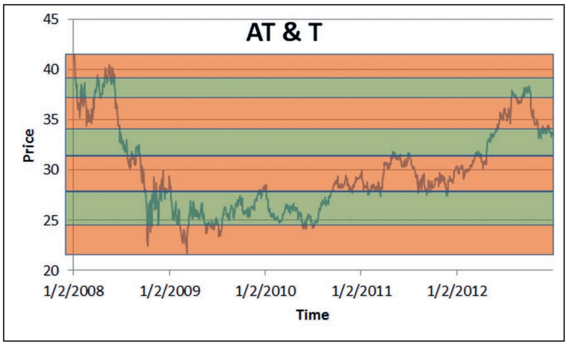

Figure 27 Fractal Segmentation, AT & T

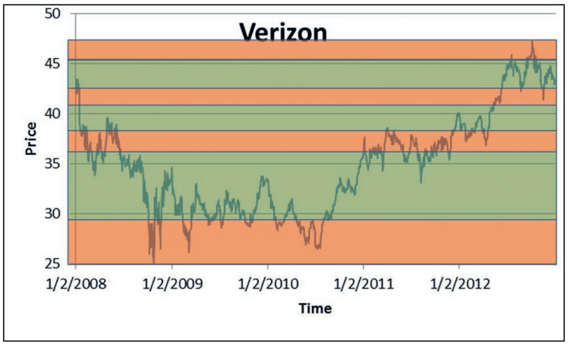

Figure 28 Fractal Segmentation, Verizon

each segment's fluctuation levels is declining. That is, the segment with the highest volatility (i.e. the segment with the steepest slope) is segment 1, followed by segment 2, and so on. Based on Cisco's price volatility analysis, the lower segment scales signify high fluctuation levels which diminish as the scale increases. On the other hand, the most volatile segment of IBM is segment 1. The other segments are ranked as follows: segment 2; segment 5; segment 3; segment 6; segment 4; and segment 7. Same with Cisco, Intel, Microsoft, and United Technology have a diminishing fluctuation as scales increases.

For AT &T, the researchers ranked the segments based on volatility levels as follows: Segment 1; Segment 2; Segment 3; Segment 5; Segment 6; Segment 4; and Segment 7. Verizon's ranking is as follows: Segment 1; Segment 2; Segment 5; Segment 3; Segment 4; Segment 6; and Segment 7. Since the main purpose of this study is to confirm the researchers' hypothesis on the fractal slope, the researchers will use the inversion formula (Equation 3) to compare the fractal spectrum with the historical price time-series. The inversion formula will be used to convert the scale value to the original value. With fractal segmentation, the advocates of the study have established the elbows of the each spectrum and translated it using the inversion formula. The identified elbow values are the threshold level of each price levels.

Each stock's time series plot is visually inspected and manually divided according to the original elbow values presented in Table 3. This is done in order to ascertain whether the volatility as determined by the fractal spectrum will be mirrored in the movements that can be seen in the time series plots. The result of this fractal segmentation is exhibited on Figures 22 to 28.

At this point of the data's interpretation, it must be remembered that the hypothesis of the fractal slope contends that the steeper the slope, the higher the volatility. However, the results of the fractal segmentation seem to negate this claim. To illustrate, CISCO has been identified to have exhibited four elbows, and the slopes of these elbows indicate that the most volatile is Segment 1. However, a visual inspection of Segment 1 in the time series plot does not clearly show a highly unstable section. To further drive the point, the slopes of IBM's elbows indicate that the three most volatile segments are Segments 1, 2, and 5. A visual representation of the segments as found in the time series plots, however, does not clearly reflect this ranking.

The researchers further note that in all the stock's examined, the ranking will always yield a result pointing to Segment 1 as the most volatile and the last segment as the least volatile. This observation led the researchers to advance the suggestion that volatility is not merely the number of times that a stock's price goes up and down. Being a measure of risk, volatility can also be associated with how rarely a particular stock price level is reached. As shown in the time series plots, all of the stocks' first segments fall under the highest price levels. These price levels, however, are rarely ever maintained. The same can be observed from the last segments representing the lowest stock price level. The number of times this stock price level occurs is relatively lesser compared to the other segments. The riskiness, then, is associated with the apparent temporariness of these stock price levels. Failure to take swift action pertinent to these stock price levels can equate to lost opportunities that are uncertain to present itself again.

5.0 Conclusion

Risk is what makes the stock market exciting. The fluctuation in stock prices is what makes it pulsate with life. Stock price changes have been a subject that captivated the attention of many, and opposing claims as to its predictability and its randomness have flourished in finance literature. The researchers noted that studying stock price volatility entails a constant regard for what can be seen in the overview and in what can be detected in the details. Quite clearly, stock price changes represent a rugged data set. This is perhaps the strongest support for the use of fractal statistics. In an attempt to measure stock price volatility, the researchers discovered that tracing the ruggedness of this data set through its fractal dimension gives a better view of the stock's behavior. Distinct volatility behaviors are recognized within the entire fluctuation level of the stock prices over time by identifying elbows within the fractal spectrum. In other words, the overview and the detail of the stock price movements both exhibit roughness of data. Thus, fractal statistics is an appropriate tool for studying this volatility.

Consequently, the researchers endeavoured to dissect this volatility into segments, proposing that the elbows of the fractal spectrum with higher slopes exhibit higher volatility. To verify this, the time series plots are divided according to the slopes determined from the fractal spectrum. The researchers conclude that visual inspection of the segments in the time series plots vis-a-vis the slopes of the elbows identified in the fractal spectrum may not be enough to draw a very clear conclusion about the volatility levels within the fluctuations in a stock price over a period of time. Owing to the vastness of the data and the estimation involved in the determination of the elbows, it may be difficult to fully notice the details within the segment simply by looking at the movements exhibited in a time series plot. The researchers believe that while fractal statistics is undoubtedly appropriate for the data set, a method to divide the fractal spectrum appropriately must first be established in order to draw out more concrete findings and conclusions.

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