Fractal Behavior of Selected Stock Market Prices

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Abstract

This research views the concept of risk in the stock market trading by examining the volatility of closing market prices through the ruggedness of information as represented by its fractal dimension. Using the historical data of the closing stock prices of McDonald's, Bank of America, and AT&T, the researchers applied fractal statistics and equated the riskiness of the stocks to their fractal behavior (i.e. fractal spectrum). An analysis of the fractal spectrum revealed a distinct behavior exhibited by the different stocks. This behavior was further explored by dividing the fractal spectrum into discrete scales that correspond to a shift in the behavior, and performing an intercompany comparison to ascertain the riskiness of the stock.

Keywords: fractals, fractals in finance, risk in the stock market, financial market, stock price volatility

JEL Classification: D7

1.0 Introduction

To look at nature with eyes that see an ironic order in the irregularities of almost everything that surrounds us —- from the trees, to the leaves, the rivers, the landscapes, even the organs of the human body —- is indeed revolutionary. This is what has made fractals such a sweeping phenomenon throughout the globe. For the most part of human existence, we have obsessed about what we consider "normal" and have been quick to dismiss some items and observations as merely isolated events and facts. Yet, a young and emerging science claims that the bigger picture that we see is actually made up of smaller replicas of itself. This posits that it is possible for us to see order in the chaos, a trend in the disorder, and a pattern in the randomness. Interestingly, the science of fractals tells us that the bigger picture is made up of smaller pictures of itself and thus, looking at merely a piece gives us an idea of the whole.

The financial market is not immune to this fractality. Finance has seen the interesting evolution from the Brownian Motion (Bachelier,1900) to Hurst exponent (Hurst, 1951) to Chaos theory and to fractals. In fact, scholars have already started recognizing the importance and benefits of using fractals in this particular area. Benoit Mandelbrot (1967), a world renowned mathematician famous for his work on fractals and the discovery of the Mandelbrot set, advanced three fractal models in finance. Fractal dimensions, together with probability distribution and two point correlation, were also utilized to analyze and measure volatility in six different stock markets (Yu, 2004). A study of the 1987 market crash used multifractal analysis that ventured into a search for identifiable patterns in the period immediately before the crash (Los, 2004). The foreign currency market has also seen their share of studies using fractals. A Multifractal Market Hypothesis has been put forward as an extension of the efficient market hypothesis in a study made by Corazza and Malliaris (2002) related to the multifractality in foreign currency markets.

Risk is a major consideration for trading in the stock market. The dips and climbs of stock prices are

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a phenomenon of curiosity in what many consider to be a game in the stock exchange arena. The Chinese pictograph for risk interestingly combines danger and opportunity, recognizing that greater risk means the opportunity to do well, as well as the danger of doing badly (Los, 2003). This is re-echoed by the maxim, "The greater the risk, the higher the reward". Indeed, the opportunity to earn in the stock market is ultimately grounded on the volatility of stock prices. A stock price which remains constant all throughout protects its investors from losses, but also denies it of gains. The volatility of the stock prices, therefore, mirrors the risk in stock trading. This research views the concept of risk in the stock market trading by examining the volatility of closing market prices expressed through its fractal dimension. Using the historical data of the closing market prices of AT&T, Bank of America, and McDonald's, this paper aims to examine risk in the context of its fractal behavior.

While it is true that fractals is a science still in its infancy, it has sparked interest from academicians and intellectuals all over the world. Finance has been one of those areas that exhibit extensive potential and benefits in the use of fractals. However, it is also to be noted that while there are already studies in the area of finance that employ fractals, it still remains a largely under-utilized discipline. As such, there could be a myriad of information out there that can be best explained by this promising view. It is with this in mind that this paper aims to contribute to the body of knowledge by specifically looking at market price changes of selected stocks using fractals.

2.0 Conceptual Framework

The significance of fractals in finance has been explored by a number of research studies. Fractal geometry has been gleaned as a technique that identifies the long term dependencies of a time series (Lento, 2009). Similarly, it has been acknowledged that, while scientific ideas related to the field of finance has been deemed to have evolved, it cannot be argued that financial models are based on rigid assumptions, mathematical formalism and methodological reductionism; thus, necessitating a more coherent approach (Velasquez, 2010). Such coherent approach includes the perspective of contemporary science, Chaos Theory, and the Science of Fractals (Velasquez, 2010). In addition, the most eminent fractal in finance is the Elliot wave model. This model identified a pattern that was found to have repeatedly and prominently appeared on the stock price charting, may it be on a small or larger scale.

Much of the appreciation on fractals, however, is confined to fractal geometry. With this observation, the researchers want to look into the behavior of selected stock prices, with a focus on its fractal dimension and its implication on the stock's riskiness. The closing market prices of three randomly selected stocks that belong to three different industries will be analyzed using fractal statistics. The objective of this study is to look at the fractal dimension (i.e. the ruggedness of the data) involving closing market prices to establish its fractal behavior. Such fractal behavior will then be used as an indicator of the stock's riskiness. The findings will supplement claims on the presence of fractals on stock price behaviour and its relationship to risk.

The researchers obtained the daily historical closing prices of the following stocks and for the inclusive years indicated herein: AT&T (January 1984 to December 2012), Bank of America (January 1986 to December 2012), and McDonalds (January 1970 to December 2012). While all three stocks were chosen randomly, they each represent a different industry, with AT & T representing telecommunications, Bank of America representing financial institutions, and McDonalds representing the fast food industry. These historical closing prices will be tested for indicators of fractal distribution. Once established, fractal statistics will be applied to determine the fractal behaviour of the stock prices through an analysis of its fractal dimension. The researchers posit that the higher the fractal dimension, the more volatile it is and the more rugged the dataset.

This volatility as measured by its fractal dimension is equated with the riskiness of the stock.

3.0 Fractal Model and Analysis of Data

Indicators of Fractal Distribution

The researchers intend to use the fractal dimension of the data gathered as a gauge for the riskiness of the stocks' behavior. In order to apply fractal statistics, the indicators for the presence of a fractal distribution must first be established. As such, the researchers generated the histograms of the closing prices of the stocks of the three companies to ascertain the absence of a normal distribution. See Figures 1 to 3.

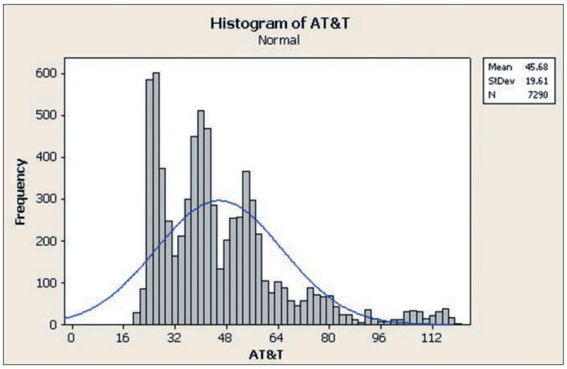

Figure 1: Histogram, AT & T

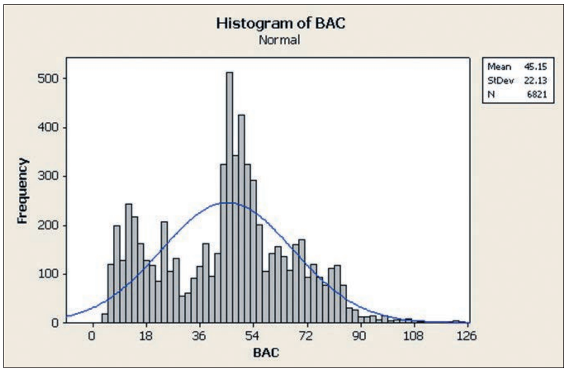

Figure 2: Histogram, Bank of America

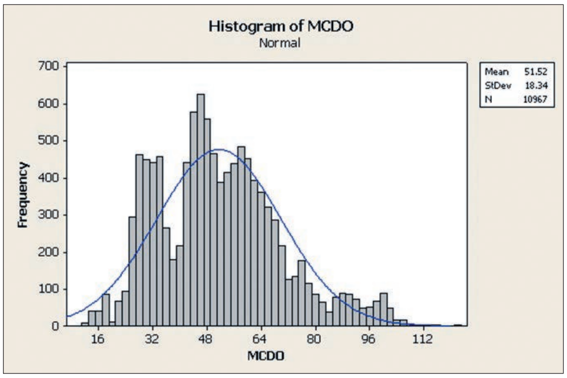

Figure 3: Histogram, McDonalds

As shown in Figures 1 to 3, the histograms of the closing prices of the stocks of AT&T, Bank of America, and Mc Donald's all indicated that they do not follow a normal distribution. The absence of a bell-shaped data allocation peculiar to a normal distribution has thus satisfied the first test for the possible presence of fractal behavior in the selected stocks. They also do not follow the shape of any other established distributions.

In addition to this ocular inspection of the histograms, the researchers further tested the data set for additional indicators. Using the Kolmogorov-Smirnov Goodness of Fit Test, the researchers performed a normality test on the data sets. The results of the normality tests are shown in Figures 4 to 6.

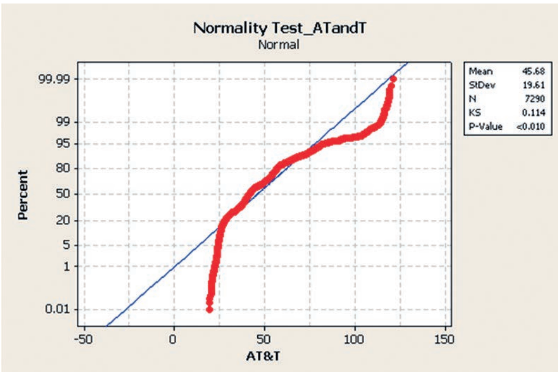

Figure 4: Normality Test, AT & T

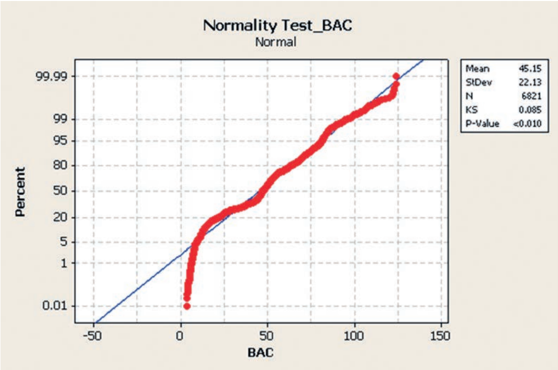

Figure 5: Normality Test, Bank of America

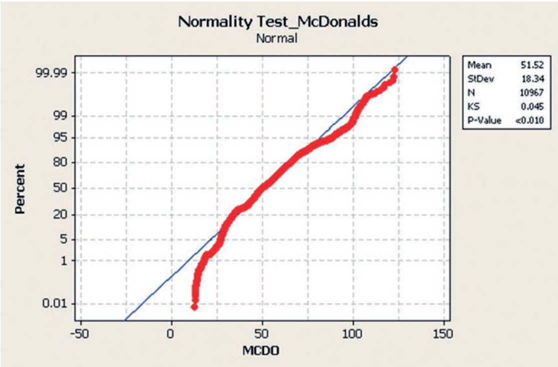

Figure 6: Normality Test, McDonalds

The normality plots in Figures 4 to 6 graphically present to us the data as plotted versus a theoretical normal distribution line. Evident in the plots are the outliers and deviations from said normal distribution line, which allow the conclusion that, as per the graphical presentation on the normality plots, it is unlikely that the data sets come from a normal distribution. Correspondingly, the *p values* of <0.01 also support the non-normality of the data. Combined with the histograms presented (Figures 1 to 3), the researchers posit that the data sets do not follow a normal distribution. As such, the researchers proceeded to apply fractal statistics.

Analysis of Data: Applying Fractal Statistics

The datasets were tabulated and ranked in preparation for analysis. Using fractal statistics proposed by Padua et al (2013), the researchers computed for the fractal dimension (λ) using the following formula:

$$\lambda = 1 - \frac{\log(1-\alpha)}{\log(\frac{x_i}{\theta})}$$

where: $\alpha = \frac{rank\ (x_i)}{n}$

and : $\theta = min\ \{x_1, x_2, \ldots, x_n\}$.

The computation of the fractal dimension (λ) is the spring board for the determination of the fractal spectrum. Barabat, Borres and Salazar (2013) propose to examine the observed data sets that are likely to have come from a fractal distribution on a spectral space instead of the data space. In the same paper it was further proposed to utilize the minimum value in the data set (θ) to serve as a powerful "microscope" that enhances the detailed picture of the observed data; thus, the value of (θ) may either be increased or decreased depending on the need to sharpen the focus on the features of the data sets. With over 6,000 data points representing closing market prices examined per stock, the researchers decided to use a uniformed minimum value (θ) of 1 for all data sets. This is done in order to promote comparability as well as to lessen the "microscopic" effect of using the smallest value in the range of daily closing prices for each stock. Table 2 shows the descriptive statistics pertinent to the fractal dimension of the closing

Table 1: Kolmogorov-Smirnov Goodness of Fit Test Results

Variable	Mean	StDev	N	KS	P-Value
AT& T	46.68	19.61	7290	0.114	<0.010
BAC	45.15	22.13	6821	0.085	<0.010
MCDO	51.52	18.34	10967	0.045	<0.010

Table 2: Descriptive Statistics of Fractal Dimension (λ)

Variable	N	Mean	StDev	Minimum	Median	Maximum
λ AT& T	7,289	1.2455	0.2171	1.0000	1.3475	2.8570
λBAC	6,820	1.2448	0.2185	1.0001	1.1805	2.8318
λMCDO	10,966	1.2395	0.2187	1.0000	1.1781	2.9369

prices of the three companies selected.

As shown in Table 2, the stock that has the highest average fractal dimension is AT&T. This is a possible indication that, among the three companies, AT&T's stock is the most risky. However, a comparison of the average fractal dimensions will not suffice to back up a conclusion regarding its riskiness. As noted, one of the key characteristics of fractal observations is that it is scale invariant and self similar. As such, after looking into the ruggedness of the data by computing for and examining its fractal dimension (λ), the scale invariance must also be established. This can be attained by defining a scale (S) measure:

$$S = \frac{1}{\log(\frac{x}{\theta})}$$

Having determined the fractal dimension and the scale measures, the researchers proceeded to establish the fractal spectrum by plotting the fractal dimensions and the scale. Figures 7 to 9 show the fractal spectrum:

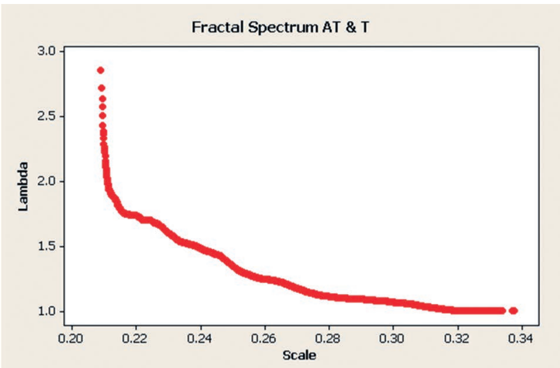

Figure 7: Fractal Spectrum - AT & T

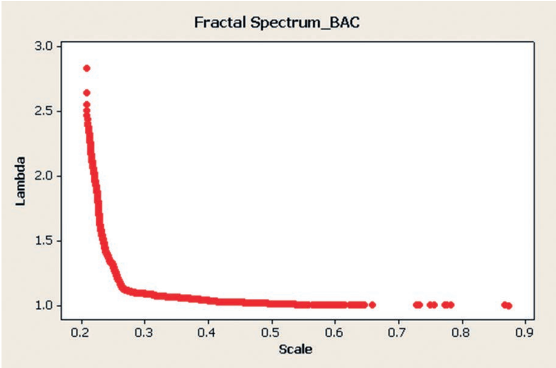

Figure 8: Fractal Spectrum - Bank of America

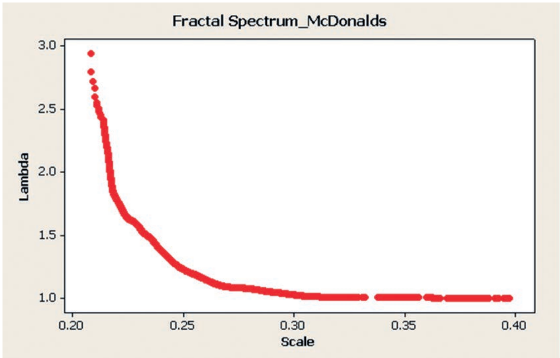

Figure 9: Fractal Spectrum - McDonald's

The fractal spectrum represents the fractal behaviour of the stocks. As noted in Figures 7 to 9, each of the three stocks behave differently. The researchers determined the peculiarities in the behavior among the three stocks by pointing out the distinct movements within the behavior it exhibits. Among the three, the most straightforward fractal behavior is that of Bank of America. As seen in Figure 8, the fractal behavior of this stock can be defined by two distinct movements (see Figure 8A). AT&T, on the other hand, shows the most shifts in its fractal behavior, demonstrating a decay that is gradual and yet irregular (see Figure 7A). The fractal spectrum of McDonald's exhibit the most neutral among the three, with a decay that is close to smooth and gradual (see Figure 9A).

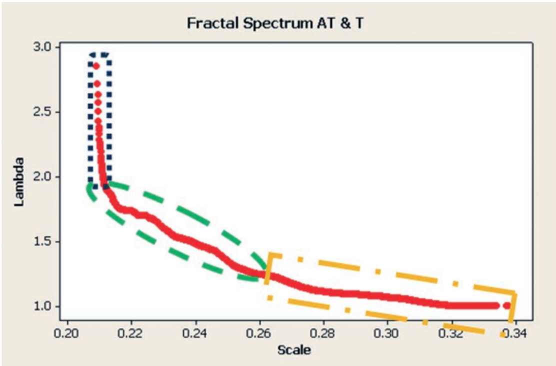

Figure 7: Fractal Behavior Movements - AT & T

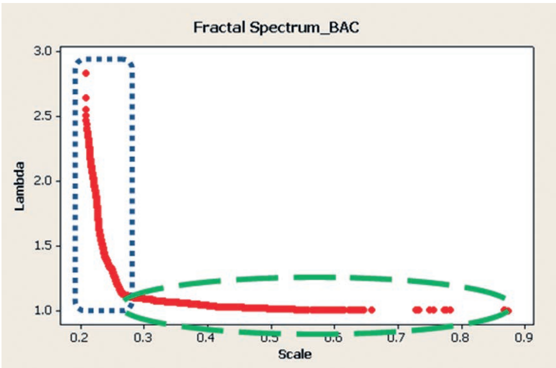

Figure 8: Fractal Behavior Movements - Bank of America

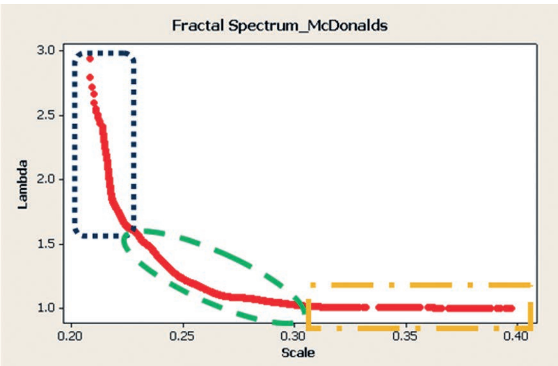

Figure 9: Fractal Behavior Movements - McDonald's

On the basis of this analysis, it can then be inferred that the more straightforward the fractal behavior, the less risky the stock. Consequently, the more irregular the decay, the more the volatility the stock's closing prices exhibit and thus, the riskier the stock. In addition to this visual appreciation of the stocks' fractal spectrum as a whole, the researchers proceeded to divide the stock's fractal behavior into small, medium, and large scales. This is done in order to look into the ruggedness or volatility of the stock prices by scale. The average value of the data belonging to the small and large scales was then raised to the power of the average of the lambda in said scales. The difference between the ruggedness in the small and large scales represent the disparity between the two scales and thus lends an insight into the riskiness of the stocks. The larger the disparity noted, the riskier the stocks. Tables 3A to 3C summarize the results of this analysis

Table 3A: Analysis per scale, AT&T

AT & T	Average of data (a)	Average of lambda (b)	a^b
Small Scale	110.9527	1.95772	10,088.04
Large Scale	32.76853	1.101894	46.76
Disparity			***10,041.28***

Table 3B: Analysis per scale, Bank of America

BAC	Average of data (a)	Average of lambda (b)	a^b
Small Scale	71.12314	1.520169	653.69
Large Scale	12.31052	1.038215	13.55
Disparity			***640.14***

Table 3C: Analysis per scale, Mc Donald's

MCDO	Average of data (a)	Average of lambda(b)	a^b
Small Scale	94.51619	1.827786	4,081.34
Large Scale	29.89889	1.041421	34.42
Disparity			***4,046.92***

Consistent with the results of the comparison of the average fractal dimension (λ) found in Table 2, and the visual representations of the fractal spectrum as shown in Figures 7 to 9, the analysis in Tables 3A to

3C also indicated that, among the three companies, AT&T's stocks tend to be the most risky. It displayed the most disparity between its small and large scales.

4.0 Conclusion

The researchers tested for fractal indicators in the data sets gathered. The histograms and normality tests indicated that the data sets comprising of the closing prices of three different stocks did not follow a normal distribution, nor did they follow any other established distributions. These results led the researchers to conclude that the data sets quite possibly exhibit a fractal distribution. The assumption of normal distribution, then, may not be appropriate. Thus, the use of traditional statistics may also not be enough in the analysis pertaining to said datasets. The researchers, after establishing the indicators of fractal distribution, employed fractal statistics by Padua (2013) to analyze the data. The general ruggedness of the information in the data sets was first examined via the determination of the average fractal dimension (λ) of each of the three stocks. After viewing the overall roughness of the information, the researchers ascertained that the stocks of AT&T exhibited the most volatility as evidenced by its high average fractal dimension as compared with the other two. The researchers then determined the fractal behavior of the stocks by examining the fractal spectrum. Similarly, it was found out that AT&T's stocks displayed the most irregular and "restless" behavior. The fractal behavior of the stocks as represented by its fractal spectrum was then dissected into small, medium, and large scales. An inspection of the disparity between its small and large scale revealed that AT&T stocks still appear to be most risky among the three.

In conclusion, the researchers posit that a stock's fractal dimension can be an indicator of its riskiness. Among the three stocks observed, AT&T has shown the most volatility in terms of its fractal dimension and its fractal behavior. This is consistent with the nature of the telecommunications industry. Being the industry most vulnerable to changes and technological advances as compared with financial institutions and fast food chains, the telecommunications industry can be viewed as inherently risky. On the other hand, the stocks of the Bank of America displayed the least volatility. This can be attributed to the strict regulations and government supervision afforded to financial institutions. In the US, for example, the Federal Reserve System, the Federal Deposit Insurance Company, the Office of the Comptroller of Currency, the Office of Thrift Supervision, and the National Credit Administrators carry out financial regulation (Ely, 2008). The degree of government supervision and regulation thus makes the stocks of the Bank of America least risky among the three companies observed.

On the basis of the results, speculative investors may give priority to investments with technological externalities. As a proposition, high variations brought by AT&T's results may generate either high capital gains or losses from price speculations. Hence, AT&T can be viewed as a high risk stock. Low variations of Bank of America, as manifested by the results, bring lower chances of capital gains and losses. Considering that the stock of the Bank of America is a low risk stock based on the results of the fractal analysis, it is less susceptible to capital gains and losses. From this, it may be inferred that the results support the maxim, "The greater the risk, the higher the reward".

To clarify the validity of the recommendations, the researchers identified one limitation of the analysis made in the study. It is not for fundamental analysis but rather highly suited for technical analysis as a conventional method in determining volatility of prices in the market. The researchers discourage analysts from utilizing the fractal statistics method in fundamental analysis. This method is better suited for technical analysis where analysts examine fluctuations and trends of prices.

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