Validation of Lepidopteran Classification by Morphological Fractal Dimension Analysis

Anna Rose P. Nillama, Mark S. Borres, Ricky B. Villeta, Roramie V. Arco, and Joyce C. Unabia

Abstract

The paper uses fractal geometry through the fractal dimension of photos of the Lepidopteran species, the commonly named butterflies, collected from the regions of Central Visayas, Philippines to determine if the morphological dimensions can discriminate one species over the other for classification purposes. The images of species were converted into a binary image using the simplest form of image processing known as thresholding effect. The binary images provide a simple view and ease for computing its fractal dimension. The fractal dimension explains the space – filling property of the image. The species available for use were the Cethosia, Idea, Pachliopta, Catopsilia, Troides where 10 subspecies of each were chosen. Also, the male species were isolated and tallied separately from the females. Empirical results revealed that the fractal dimensions can differentiate the species from one another with a general p – value of 0.000 and F – value of 6.483. All sample pairwise comparisons were tested for the significant differences and still differentiates one over the other as evidenced by the corresponding p - values of all equal to 0.000, respectively. Moreover, the fractal dimensions, in relation to the species gender, were tested for possible significant differences, and findings showed that it could differentiate across butterfly's gender with a p – value of 0.000 and F- value of 8.037. The empirical probability of wrong classification using the fractal dimension is less than 1%.

Keywords: lepidoptera species, fractal geometry, fractal dimension, morphological dimensions and tresholding effect

1.0 Introduction

Lepidoptera (moths and butterflies) is the second largest order in the class Insecta. The butterfly is a diverse insect, found in many colors and sizes. Adults have large, often covered with minute overlapping scales and produces a brightly colored wing pattern that plays an important role in courtship and intraspecific identification. They are conspicuous.

Butterfly plays an important responsibility in ecosystems, as it acts as a pollinator, a source of food and an indicator of the ecosystem's being (Kumar, 2013). Therefore, an abundance of butterflies usually indicates a healthier ecosystem. They move pollen grains during migration over a distance improving genetic diversity in the plant species. Where habitats are larger and less isolated, the species breed over a very wide area and exploit a wide range of habitat types and new species are then discovered but according to Shashikumar (2013) due to unscientific management of natural resources, much of our native butterflies are fast disappearing. Owing to various reasons such as habitat destruction for development (homes and other infrastructure), scarcity of both larval and adult food plants, their population may be severely affected in the near future.

As far as the Lepidoptera are concerned, problems in the higher classification of sorting individuals into species categories is not new; several ways are to be discussed and to be agreed that even the adoption of the Linnaean system has not proved an unqualified success. De Jong (1996) stated that the progress in understanding the higher classification of butterflies has not kept pace with the increase number of described species. Important points of uncertainty or contention include, ranking problems and the division of families into subfamilies. On the other hand, Knowlton and Jackson (1994) state that establishing techniques that provide accurate characterization of species and that minimize ambiguity in allocating individuals to species groups is important.

The study implies the use of fractal dimensions as bases for the butterfly species taxonomic classification since fractal is very sensitive in measuring and detecting small changes in the insect's morphology.

Benoit Mandelbrot first used the term fractal to designate objects that are self-similar at different scales, and the fractal concept provides a useful tool to explain a variety of naturally occurring phenomenon (Lapinig, et.al., 2013). A fractal is an irregular geometric object with an infinite nesting of structure at all scales. Fractal objects can be found everywhere in nature such as coastlines, fern trees, snowflakes, clouds, mountains, and bacteria. Some of the most important properties of fractals include self-similarity, chaos, and non-integer fractal dimension. Fractals are self-similar, which means that structures are repeated at different scales of size (Peng, et al., 2005). When a structure is fractal in nature, it can describe with simple and iterative rules, no matter how complex it may seem by Euclidean standards (Chang 1993; Briggs & Peat 1989).

Analysis using fractal dimension can be a very powerful tool in solving important problems in the misclassifications of species specially those species that grows rapidly. It improves the accuracy in describing and classifying objects in nature thus, the need for further studies of fractals would be of great help to provide a reliable and accurate characterization of species.

In this paper, we identify and classify selected species of Lepidoptera in terms of genus differentiation and gender classification using fractal geometry through the fractal dimensions of photos of the Lepidopteran species, commonly named butterflies, which are collected from the regions of the Central Visayas, Philippines.

2.0 Related Literature

Binomial system of nomenclature was first proposed by the Swedish naturalist Carolus Linnaeus (Carl von Linne), who introduced it in the tenth edition of his Systema Naturae in 1758 (Corbet and Pendelbury, 1934). According to the Linnaean scheme, all animals and plants are known by a combination of two names, the first indicating the genus and the second the species. It is not easy to define this latter term precisely, but, for the immediate purpose, a species may be regarded as an assemblage of individuals closely resembling one another and of common descendant, sharing certain distinctive characters and having a common area of distribution, and potentially capable of interbreeding.

On the other hand, Adrian Hoskins states that taxonomy is not an exact science. It relies on suppositions about the relationships between various species, and about their evolutionary ancestry. Because of this, the classification (and the scientific names) of butterflies and moths are under constant revision. Sometimes, it is a simple matter of relocating a species to a different genus. On rare occasions however, it may require that a genus, tribe or even an entire subfamily be migrated from its familiar home and relocated into another family or superfamily. The inexact nature of taxonomy, and the fact that it relies on opinion rather than fact. Unfortunately, scientists often find themselves in disagreement about which genus a particular species belongs, or whether two or more butterflies with very similar characteristics should be classified as separate species or as subspecies.

Recent advances in biochemical and electron microscopic techniques, as well as in tests investigating the genetic relatedness among species, have redefined previously established taxonomic relationships (Cain, 2014). With the advent of modern molecular biology, the amount of DNA sequence data from diverse taxa has exploded. This advance has opened new avenues which is to analyze the relationships among organisms. In contrast to anatomy, which is largely a qualitative analysis, DNA and protein comparisons permit the construction of quantitative comparisons among creatures. Furthermore, since DNA is the molecular basis of heredity and is, in a sense, a record of a creature's genetic ancestors, modern molecular biology allows the direct assessment of an organism's genealogy. Together, these advances have resulted--and are continually resulting--in an enormous increase in data that have yet to be fully fitted into a classification scheme (Jeanson, 2010).

In Biology, organisms trace a path that is known to be fractal and generally obeys the fractal dimension of its ecosystem. This observation is very important in landscape ecology. Likewise, the distribution patterns of biological organisms across various trophic levels are fractals e.g. more at the primary producer level than at the top consumer level. The life cycles of organisms (insects to mammals) are also observed to be fractal (Borres, 2013).

More generally, fractals have been used successfully in various other fields. Spatz (2013) used fractal geometry for a faster and more reliable method of determining cancer cell types very accurately. Dean, et al. (2013) observed a rare quantum effect that produces a repeating butterfly-shaped energy spectrum, confirming the long standing prediction of this quantum fractal energy structure, called Hofstadter's butterfly; Perfect and Kay (1995) applied fractals in the soil's properties, physical processes and spatial variability; Okubo (2012) used fractal geometry in the San Andres Fault System. It is evident that fractals will represent the future of many areas of sciences, especially in biology (Chang, 1993).

With regards to the existing butterfly species in Visayas Region, Philipines, Semper 1888, described that Cethosia luzonica boholica can be found in Central Visayas, particularly in Bohol and Cebu as the species' name suggests. Cethosia luzonica boholica is under the family Nymphalidae.

The commonly known "paper kite" or "large tree nymph" butterfly, Idea leuconoe obscura was first described by Staudinger last 1889. It is under Nymphalidae family. It is also known especially for its presence in butterfly greenhouses and originated in Southeast Asian countries. The larvae feed on Parsonsia sp. like P. helicandra and P. spiralis. They can be found in Bohol, Negros, Leyte, and Dinagat.

Catopsilia pomona pomona is one of the commonest butterflies that can be found all throughout the Philippines except in the deepest forest. They are commonly known as the "lemon emigrant" under the family of Pieridae. They love to sip the nectars of the flowers in Lantana plants in the gardens. There have been some experiments done to try and explain the frequency of the different foms, and there is some justification in suggesting that the length of daylight hours may have a part to play, but it is unlikely to be the only factor. Population density and temperature may also be of relevance. This butterfly displays a strong migratory tendency. Two male forms and five female forms were discovered by polymorphism. The males are easily identifiable, even in flight, they can sometimes be found "puddling" or congregating at moist spots on roadsides or riverbanks to imbibe precious minerals not found from nectar. Identifying females is harder though not impossible, due to their many different colour forms. Known food plants or host plants include Cassia fistula, Cassia siamea and Butea frondosa.

Pachliopta phegeus was originally described as Papilio Art phegeus by Hopffer in 1866. They can be found in Southern Philippines (Mindanao, Samar, Leyte, Bohol and Cebu). They are not rare and not considered to be threatened. It is believed to be synonym with Pachliopta leytensis. This species of butterfly is under Papilionidae.

One of the most striking butterflies is the "Golden Birdwing", Troides rhadamantus birdwings are named for their birdlike flight, their angular wings sometimes with featherlike patterns. The first description was in 1835 by Lucas. Is under Papilionidae. They possessed thermoreceptors that are sensitive to sudden increase in temperature. They are fairly distinctive since it is confined to the Philippines where it is guite widespread and generally very common. Many subspecies can be found on different islands. Its host plant is Aristolochiagrows - including public and private gardens. Its relationship with Troides rhadamanus rhadamantus is unsettled. Some taxonomists (Haugum & Low (1985) argue strongly for placing both plateni and dohertyi as subspecies of T. rhadamantus, while others (Tsukada 1980; Nishiyama 1982) accept all of them as distinct species. The wingspan is about 14 – 16 cm. This butterfly is a member of the family Papilionidae. Troides rhadamantus is black. The hind wings are golden. The females are dark-brown and larger than the males. They can be found in Luzon, Mindoro, Marinduque, Negros, Samar, Masbate, Romblon, Polillo, Panay, Cebu, Leyte, Bohol, Dinagat, Mindanao, Bazilan, Jolo, Tawitawi, Guimaras, Panaon and Camiguin.

A study of butterflies has highlighted a hotly debated glitch in DNA barcoding, a scheme by which some researchers hope to quickly catalogue vast numbers of species. Biologists sampling Karner blue butterflies have found that genetic scans failed to identify the endangered animals.

3.0 Design and Methodology

This study utilized a descriptive and comparative method of research, which aimed to apply a novel approach in validating the categorization of previously identified Lepidopteran species to their respective classification using fractal geometry through fractal dimension.

As to site location, the researcher randomly collected different species of butterfly from the Jumalon Museum, Butterfly Sanctuary and Art Gallery. It is a private museum, art gallery and nature reserves run by the Jumalon Foundation. It is located in Julian N. Jumalon St. Basak, Pardo, Cebu City, Philippines.

For the selection process, a total of five butterfly species each having five males and females per sample were gathered. These butterflies can only be found in Central Visayas, Philippines. Due to resource limitations, the researcher used the available specimens in the sanctuary.

A digital single-lens reflex camera (DSLR) was then used to capture the detailed image or the scale and edges of the butterfly for a fixed distance of 1 ft. (12 inches) from the platform. Digital Single Lens Reflex camera has large image sensors that produce high-quality photos, that the overall quality and response is the biggest advantage. It has built-in mirrors (reflex) so the image seen bounces up to the viewfinder. One of the best features of a DSLR is the ability to switch lenses. The majority of compact digicams cannot match the quality of a DSLR since imaging sensors are much larger. The bigger the sensor, the better the photograph (Benson, 2013).

Each butterfly was then carefully mounted over a white platform and the room lighting was controlled in order to ensure that there will be no shadow and only the specimen's features are caught on camera. The resulting images were converted into a binary image using the simplest form of image processing known as thresholding effect. The binary image provided a simple view and ease for computing its fractal dimension.

Each butterfly image generated was then inputted in the FRAKOUT software to calculate their fractal dimensions.

Concept of a Fractal and Fractal Dimensions

Classical geometry considers objects that have integral dimensions: points have zero dimension, lines have one dimension, planes have two dimensions and cubes have three dimensions. Within a plane, one can represent points and straight lines and other geometric objects as shown below:

It is possible to represent geometric objects



PAADAARSS

within a plane that are neither points nor lines like the squiggly line above. This squiggly geometric object cannot have dimension equal to 1 because it fills up more space than a line; it cannot have dimension equal to 2 because it does not form an area. Hence, its dimension λ has to be between 1 and 2 like $\lambda = 1.63$. We will say that the squiggly line is a fractal (a geometric object having fractional dimension).

The fractal dimension of an object defines its roughness, ruggedness or fragmentation. The higher the fractal dimension, the more rugged and irregular-looking is the object. Thus, although fractals are rough and irregular objects, the pattern of irregularities are repeated over and over again. This is called the self-similarity property of fractal. Benoit Mandelbrot (1967) is acknowledged as the mathematician who opened roughness as a legitimate topic for investigation in modern science. He claimed that nature and natural processes are fractals, while uniform, smooth and continuous patterns are man-made concepts and pervade mathematical analysis. He also said that by introducing "randomness" into the situation, one gets more realistic fractal representations.

After the publication of Mandelbrot's book: Fractals: The Geometry of Nature, many scientists used fractals with great success (Cohen, 1987) on fractal antennae; (Krummel et al., 1987) on forest fractals and others). It has found applications in various disciplines as well as in many areas of practical technology.

In Padua (2012), fractal geometry was translated to statistical language. A probability distribution akin to Pareto's distribution for incomes was proposed as a model for fractal random variables X:

⁽¹⁾
$$f(x) = \frac{(\lambda - 1)}{\theta} \left(\frac{x}{\theta}\right)^{-\lambda}, x \ge \theta, \lambda > 0$$

Where λ = fractal dimension of x, . A maximum – likelihood estimator for λ based on a random sample of size n was provided as:

(2)
$$\hat{\lambda} = 1 + n \left(\sum_{i=1}^{n} log \left(\frac{x_i}{\theta} \right) \right)^{-1}$$

Then proceeded to show that for n=1:

(3)
$$z = \hat{\lambda} \log\left(\frac{x}{\theta}\right) - 1 \underset{\sim}{d} Exp(\lambda - 1)$$
 or:

(4)
$$q(z) = (\lambda - 1) \exp(-(\lambda - 1)z)$$

For a random sample of size n, the random variable:

(5) $q = \hat{\lambda} \sum_{i=1}^{n} \log\left(\frac{x_i}{\theta}\right) - n$

Has the same $q^* = \sum_{i=1}^n \log\left(\frac{x_i}{\theta}\right) = \sum_{i=1}^n Z_i$ distribution as . The distribution of (5) is therefore $Gamma\left(n, \beta = \frac{1}{\lambda-1}\right)$ where $\lambda > 1$:

(6)
$$h(q) = \frac{(\lambda-1)^n}{\Gamma(n)} q^{n-1} e^{-q(\lambda-1)}$$

$$q > 0, \Lambda > 1$$

 $h(q) = \frac{(\lambda - 1)^n}{\Gamma(n)!} q^{n-1} e^{-q(\lambda - 1)}$

Thus, if we have one sample of a species and if we are able to estimate its (geometric) fractal (see for example some available freeware like FRAK. OUT), then we are able to compare the fractal dimension for species (say, λ_1) with the specimen (λ_2):

(7)
$$u = |\lambda_1 - \lambda_2|$$

We approximate the distribution of u by an exponential distribution and obtain:

(8)
$$\delta_s = P(u \ge \varepsilon) = \frac{1}{2} \left(1 + exp(-\varepsilon^{(\lambda_2 - 1)}) \right)$$
, a similarity index

where $\lambda_{2=}$ fractal dimension q specimen species. We refer to (8) as a similarity index. As the difference $\varepsilon = |\lambda_1 - \lambda_2|$ increases, the similarity index decreases. If $\lambda_1 = \lambda_2$ (hence, λ_1), the fractal dimensions are identical and the two documents are 100% similar. This means that the two species contains exactly the same fractal characteristics: straight lines, curves, strokes, spacings, slants and so on, and, must therefore belong to the same species.

It is also possible to determine what values of λ_2 will yield high similarity index thus:

$$(9) \quad \lambda_{2} \varepsilon = |\lambda_{1} - \lambda_{2}|$$

For instance, if $\lambda_1 = \lambda_2$, then the values of above will indicate 95% similarity index or greater.

The percent correct identification is then calculated as:

$$\lambda_2 \quad u = |\lambda_1 - \lambda_2|$$

The PCI's are then compared across species to determine if the proposed methodology is sensitive to species differences through an analysis of variance methodology.

To augment the metric (10), we also computed the average similarity index (λ_2) per species:

 $\varepsilon = |\lambda_1 - \lambda_2|$

Furthermore, the One – Way Analysis of Variance (ANOVA) was used to Identify the factors that influenced a given data set and the significant differences between the individual species and gender

4.0 Results and Discussion

4.1 Lepidopteran species in the collection

The following are the images of lepidopteran species in the collection that have undergone fractal dimension analysis. Gender differentiation was described according to the expert in the sanctuary.



4.2 Classification using fractal geometry in terms of genus differentiation and gender



No. of Trials	Cethosia Idea leuconoe Iuzonica boholica obscura		uconoe cura	Catopsilic pom	a pomona nona	Pachliopta phegeus		Troides rhadamantus		
	Female	Male	Female	Male	Female	Male	Female	Male	Female	Male
Trial 1	1.8628	1.8522	1.8517	1.8478	1.8268	1.7799	1.8352	1.8152	1.8484	1.8394
Trial 2	1.8629	1.8516	1.8538	1.8481	1.7994	1.7751	1.8347	1.8145	1.8483	1.8355
Trial 3	1.8634	1.8549	1.8544	1.8480	1.8029	1.7801	1.8345	1.8148	1.8487	1.8354
Trial 4	1.8631	1.8515	1.8543	1.8481	1.8027	1.7778	1.8346	1.8150	1.8489	1.8358
Trial 5	1.8627	1.8548	1.8541	1.8469	1.8032	1.7801	1.8344	1.8146	1.8487	1.8357
Average fractal dimension(each gender)	1.8630	1.8530	1.8537	1.8478	1.8070	1.7786	1.8347	1.8148	1.8486	1.8364
Average fractal dimension(each species)	1.8580		1.8	507	1.79	928	1.8	247	1.84	125

Table 1: Summary of Empirical Fractal Dimensions of Lepidoptera

After conducting the five trials, the results were tabulated to compare the differences of the female and male dimensions in each species. Cethosia luzonica boholica has the highest fractal dimension of 1.8580 among the five species of butterflies followed by Idea leuconoe obscura which has a fractal dimension of 1.8507. Catopsilia pomona pomona has the lowest fractal dimension of 1.7928 among the five species. Pachliopta phegeus has the average fractal dimension of 1.8247 while Troides rhadamantus has 1.8425. All the butterfly species displayed high fractal dimensions which demonstrated that these butterflies were indeed highly complex and rugged in shape and form. The fractal dimension of an object defines its roughness, ruggedness or fragmentation. The higher the fractal dimension, the more rugged and irregular-looking is the object (Lapinig, 2013). Cethosia luzonica boholica has the most complex and rugged features among the five butterfly species (Table 1). On the other hand, Catopsilia pomona pomona registered the lowest fractal dimension which means that the species was the less rugged among the five (5) butterfly species.

The figure below (Fig. 7) is a graphical outline of Table 1 which illustrates the average from the fractal dimensions of five females and five males in each species. Cethosia luzonica boholica (1.8580) is monotonically higher among the five species. It was then followed by Idea leuconoe obscura (1.8507). Catopsilia pomona pomona (1.7928) has the lowest fractal dimension as reflected by the graph. The result indicated the level of ruggedness or roughness of an organism. The higher the dimension the more rugged is the object. Figure 8 shows that females have high fractal dimensions compared to males and this is true to all the butterfly species collected. According to Williams (2009), generally, female butterflies were larger than males. This is for reproductive purposes, as females need larger abdomens and wings in order to carry their eggs, which number in the hundreds per clutch.







Figure 8: Graph of the Fractal Dimensions of the five butterfly species in accordance to its gender.

Table 2: One–Way ANOVA: Cethosia luzonica boholica, Idea leuconoe obscura, Catopsilia luzonica boholica, Pachliopta phegeus, Troides rhadamantus

Source of Variation	SS	df	MS	F	P-value	
Between Groups	0.027	4	0.007	t6.483	0.000	
Within Groups	0.047	45	0.001			
Total	0.074	49				

ANOVA: Single Factor

Table 3: One way ANOVA of the different species according to its genders.

Source of Variation	SS	Df	MS	F	P-value	
Between Groups	0.018	4	0.005	8.037	0.000	
Within Groups	0.011	20	0.001			
Total	0.299	24				

The comparison of the fractal dimensions between butterfly species using One - Way - ANOVA revealed that the fractal dimensions of the five (5) butterfly species are significantly different (F=6.48, p=0.000). The high computed F-value could be attributed to the small standard errors of the mean (fractal dimension) computed for each species. This means that butterflies belonging to the same species have relatively the same fractal dimension in comparison to butterflies belonging to different species. The similarity index computed for the fractal dimensions of the butterflies exceeded 99%.

Further analysis showed that fractal dimensions can differentiate across the gender of the butterfly and thus, significantly different (F=8.037, p=0.000). That is, the proposed methodology using fractals in classifying the butterfly species across genus and gender would be able to detect differences. On the other hand, all the butterfly species could be easily detected by the proposed methodology and was very effective.

Pair	Mean Difference	<i>p</i> – value
Cethosia vs. Idea	0.0073	0.000
Cethosia vs. Catopsilia	0.0652	0.000
Cethosia vs. Pachliopta	0.0333	0.000
Cethosia vs. Troides	0.0155	0.000
ldea vs. Catopsilia	0.0579	0.000
Idea vs. Pachliopta	0.0260	0.000
Idea vs. Troides	0.0082	0.000
Catopsilia vs.Pachliopta	-0.0319	0.000
Catopsilia vs. Troides	-0.0497	0.000
Pachliopta vs. Troides	-0.0178	0.000

After conducting the five trials, the results were tabulated to compare the differences of the female and male dimensions in each species. Cethosia luzonica boholica has the highest fractal dimension of 1.8580 among the five species of butterflies followed by Idea leuconoe obscura which has a fractal dimension of 1.8507. Catopsilia pomona pomona has the lowest fractal dimension of 1.7928 among the five species. Pachliopta phegeus has the average fractal dimension of 1.8247 while Troides rhadamantus has 1.8425. All the butterfly species displayed high fractal dimensions which demonstrated that these butterflies were indeed highly complex and rugged in shape and form. The fractal dimension of an object defines its roughness, ruggedness or fragmentation. The higher the fractal dimension, the more rugged and irregular-looking is the object (Lapinig, 2013). Cethosia luzonica boholica has the most complex and rugged features among the five butterfly species (Table 1). On the other hand, Catopsilia pomona pomona registered the lowest fractal

Table 5: Summary of Mean difference and p-values	comparing the fractal	dimensions between g	enders
--	-----------------------	----------------------	--------

Name of species	Female vs. Male Mean Difference	<i>p</i> -value
Cethosia luzonica boholica	0.0100	0.000
Idea leuconoe obscura	0.0059	0.000
Catopsiliapomona Pomona	0.0284	0.000
Pachliopta phegeus	0.0199	0.000
Troides rhadamantus	0.0122	0.000

June

Mean difference between genders (Table 5) of Catopsilia pomona pomona has the highest mean difference of 0.0284 followed by the female vs. male of Pachliopta phegeus with the mean difference of 0.0199. On one hand, Troides rhadamantus and Cethosia luzonica boholica have the mean differences of 0.0122 and 0.0100 respectively while Idea leuconoe obscura, reflected the lowest mean difference of 0.0059.

Tables 5 and 2 revealed empirical results that the fractal dimensions can differentiate the genus of species from one another with a general p-value of 0.000 for an F-value of 6.48. All sample pairewise comparisons were tested for significant differences and still differentiates one over the other with p-values of all equal to 0.000, respectively. The fractal dimensions according to the species gender were likewise tested for significant differences and findings deduced that it can differentiate across butterfly's gender with p-value of 0.000, and F-value 8.037. The empirical probability of wrong classification using the fractal dimension analysis is less than 1%.

5.0 Conclusions

Using fractal dimension analysis in this study was a potentially powerful technique in validating butterflies according to their appropriate genus and gender yielding a low misclassification probability of 1% or less. Moreover, butterflies belonging to the same species have equally high similarity index exceeding 99% which means that this method can also be used in classifying and allocating organisms aside from using the early ways of classifying butterflies.

References

Books

- Baltazar, C. R. (1991). An inventory of Philippine Insects II. Order Lepidoptera (Rhopalocera). -399 pp., Los Banos, Philippines (College of Agriculture, University of the Philippines.)
- Briggs, J. & Peat, D. F. (1989). Turbulent Mirror. New York: Harper & Row, Publishers, Inc.
- Corbet, A. S. & Pendlebury, H.M. (1992). The Butterflies of the Malay Peninsula. Kuala Lumpur. United Selangor Press Sdn. Bhd.
- Smart, P. (1976). Encyclopedia of the Butterfly World. Hamlyn Publishing Group Ltd.
- Tsukada, E., (1980). Butterflies of the South East Asian Islands, I, Papilionidae.-51, 91, 230-231 pp., 277 incl. 27, 63 pls., Plapac, Japan.
- Tsukada, E., (1981). Butterflies of the South East Asian Islands, II, Pieridae. Danaidae. 47-49, 153-157, 262-264, 518-520 pp., incl. 15-17, 113-117 pls., Plapac, Japan.
- Tsukada, E., (1985). Butterflies of the South East Asian Islands, IV. Nymphalidae (I).-75, 292 pp., incl. 43 pls., Plapac, Japan.

Journals

- Borres, M. (2013). From Fractal Geometry to Statistical Fractal.
- Lapinig, V. T., Almirol, C. C., Sabandal, M. O., Gov. Alfonso D. Tan College, Mindanao University of Science and Technology, Northwestern Mindanao State College, Tangub City (2013), Classification of Mangrove Species Based on Leaf Fractal Dimensions.
- Padua, R.; Palompon, D.; Onton, D. (2012). Data Roughness and Fractal Statistics (CNU Journal of Higher Education Research, CHED-JAS Category A, Vol. 7, no. 2

Literatures Cited

- Benson, T. (2013, November). Digital Trends: What is DSLR? It's the camera that raises you from amateur to advanced hobbyist. Retrieved from http://www. digitaltrends.com/photography/what-is-a-dslr/
- Cain, A.J. (2014). A classification of living organisms. Encyclopedia Britannica. Retrieved from http:// www.britannica.com/EBchecked/topic/584695/ taxonomy/48704/A-classification-of-livingorganisms
- Chang, A. (1993, February). Fractals in Biological Systems. Retrieved from http://poignance.coiraweb.com/ math/Fractals/FractBio/FractBio.html
- Dean, C. R., et al. (2013, May). Hofstadter's butterfly and the fractal quantum Hall effect in moiré superlattices. Nature International weekly journal of Science. Retrieved from http://www.eurekalert. org/pub_releases/2013-05/cu-fdp051313.php
- Jeanson, N. T. (2010). New Frontiers in Animal Classification. Retrieved from http://www.icr.org/ article/new-frontiers-animal-classification/
- Knowlton, N., and J. B. C. Jackson. (1994). New taxonomy and niche partitioning on coral reeds – jack of all trades or master of some. (Trends Ecol. Evol. 9: 7-9)
- Kumar, S. (February 2013). Role of Butterflies in the Ecosystem. Retrieved from http:// justbefirsttoknow.blogspot.com/2013/02/role-ofbutterflies-in-ecosystem.html
- Okubo, P. (2012). Fractal Geometry in the San Andres Fault System. Journal of Geophysical Research: Solid Earth. 92, 345-355. DOI: 10.1029/ JB092iB01p00345
- Peng, F, Yu, X, Xu, G., and Xia, Q. (2005, NOvember). Fuzzy Classification Based on Fractal Features for Undersea Image. International Journal of Information Technology, Vol. 11. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?d oi=10.1.1.97.6264&rep=rep1&type=pdf

- Perfect, E. & Kay, B. D. (1995, November). Applications of fractals in soil and tillage research: a review. Soil and Tillage Research, 36 (1-2), 1-20. Retrieved from http://www.sciencedirect.com/science/ article/pii/0167198796813973#
- Shashikumar, L. (2013, March). Butterflies reflect health of ecosystem. Retrieved from http://www. deccanherald.com/content/316469/butterfliesreflect-health-ecosystem.html
- Spatz, J. (2013, December). The geometry of cancer cells. Max-Planck-Gesellschaft. Retrieved from http:// www.mpg.de/7647926/cancer_cell_fractal
- Williams, A. (2009). How to tell the gender of the butterfly. Retrieved from http://animals.pawnation.com/ tell-gender-butterfly-6608.html