Societal Offenders In Prison: Global In-Country Analysis By Fractals

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Abstract

This study examines the relationship between a country's human development index (HDI) and the number of societal offenders in jail (Off Index) for the country. Traditional analysis of this nature strongly veers towards linear (regression) analysis which may be inappropriate for the present case because of the high variabilities viz. fractality, of the data for the number of offenders in jails. The fractal correlation coefficient (fractogram) and the correlation of the fractal dimensions in tandem give a clearer picture of the degree of relationship induced by one highly variant variable on another variable than using the ordinary correlation coefficient. This is illustrated in the case of the relationship between the Human Development Index and the number of incarcerated offenders in the various countries of the world.

Keywords: fractogram, fractal correlation, human development index,
      incarcerated offenders

1.0 Introduction

The prisoners in jails represent the percentage of population who are considered unfit to join society either because they pose danger or because society believes that their seclusion from the normal flow of life would provide them with the necessary time to rehabilitate themselves and thereafter become productive members of again. The growing number of prisoners in jails across different countries in the world is alarming (United Nations Office on Drugs and Crime , 2002) and requires a closer look into the phenomenon in order to arrest this trend. This study examines the relationship between a country's human development index (HDI) and the number of societal offenders in jail (Off. Index) for the country. Traditional analysis of this nature strongly veers towards linear (regression) analysis which may be inappropriate for the present case because of the high variabilities viz. fractality, of the data for the number of offenders in jails. We propose to analyze the phenomenon using fractogram or fractal correlation analysis.

Fractal correlation analysis is a relatively new addition to the arsenal of statistical techniques to analyzing relationships that may not be linear in form. In the past, linear analysis of the relationship between the number of jailed individuals and aspects of human development had been done (Max Stevens, 2008). The high proportion of prisoners in developed countries may be explained by a range of factors, including better funded criminal justice systems, a more strict approach to law and order (e.g. through the use of mandatory sentencing), and a larger gap between the rich and the poor. In non-developed countries, rates

of incarceration may be a reflection of a tendency for some crimes to go unpunished, political corruption, or the use of other mechanisms which provide an alternative to incarceration as a means of dealing with crime (e.g. through the use of reconciliation). These factors all have relationship to the dimensions of Human Development Index.

The Founder of the Human Development Report Mahbub ul Haq (1998) averred "The basic purpose of development is to enlarge people's choices. In principle, these choices can be infinite and can change over time. People often value achievements that do not show up at all, or not immediately, in income or growth figures: greater access to knowledge, better nutrition and health services, more secure livelihoods, security against crime and physical violence, satisfying leisure hours, political and cultural freedoms and sense of participation in community activities. The objective of development is to create an enabling environment for people to enjoy long, healthy and creative lives." Thus, the Human Development Index developed over the years covers the main dimensions of education, health and income which are indicated by educational attainment, mean years of schooling, expected years of schooling, life expectancy at birth and gross national income per capita respectively. (Human Development Report Office, UNDP, 2013).

While it is clear that the rate of incarceration varies inversely as the nation's human development index (HDI), a straight correlation analysis reveals a non-significant correlation coefficient of 0.118 (p-value= 0.138). Not only is this figure misleading in terms of the direction of the relationship, but more importantly it implies that human development has nothing to do with the rate of incarceration or the population of societal offenders. Obviously, a different kind of analysis is required in order to better describe the actual situation obtaining in the various countries. This is the purpose of the present study.

2.0 Basic Concepts in Fractal Statistics

Fractal statistical analysis applies to situations where the mean or first moment does not exist. It also applies to situations where smaller fluctuations dominate the larger ones. Padua (2012) suggested using a power law distribution similar to Pareto's distribution given by:

$$1....f(x) = \quad , \lambda > 0, \theta > 0, x \geq \theta$$

where λ is defined as the fractal dimension of X and θ is the smallest (positive) value of the random variable.

The maximum likelihood estimator of λ is:

$$2.... \ \hat{\lambda} = 1 + \frac{1}{\log(\frac{x}{\theta})}$$

so that each observation contributes to the fragmentation of the support X. Padua (2013) demonstrated that the distribution of the maximum likelihood estimators obey an exponential type of distribution so that both the mean and variance of the fractal dimensions exist.

A device called fractal spectrum or $\lambda(s)$ spectrum was suggested by Padua et al.(2013) to identify locations on the support X where high data roughness or fragmentation occur and where smoothness appear to dominate. The spectrum is defined as:

$$3..... \ \lambda(s) = 1 - \frac{\log(1-\alpha)}{s} = 1 - \frac{\log(1-\alpha)}{s}$$

where $X\alpha$ is the αth percentile of X and

$$s = \log\left(\frac{x}{\theta}\right)$$

Deviations from smoothness indicate the severity of poverty incidence in a given context. A test for deviation from smoothness i.e. H0 :$\lambda = 1$, is suggested in the second paper of Padua (2012) and the reader is referred to the paper as provided in the list of references.

3.0 Research Designs and Methods

The study is descriptive in nature and aims to validate a new procedure for assessing relationships between two variables that do not behave as realizations from a normal distribution but from a power law or fractal distribution. Data for the variables are obtained from http://www.nationmaster.com/graph/cri_pri-crime-prisoners for the incarceration statistics and www.epw.in/.../human-development-index-trends-1980-2012 for the HDI of the countries of the world.

The data obtained are analyzed first by utilizing statistical software to determine the one-dimensional representations of the incarceration rates and the human development indices. This one-dimensional graphical representation was then exported to a fractal software available for free in the net. The fractal software outputs the fractal dimensions of the variables in question. In turn, these fractal dimensions represent the degree to which the variables fragment a smooth straight line.

The two-dimensional scatterplot of the number of offenders incarcerated versus the human development indices of the countries was plotted using the same statistical software. Once again, the plot was exported to the fractal software to obtain the resulting fractal dimension of the two-dimensional graphical representation. This fractal dimension now represents the effect of the ruggedness or roughness of the figures representing Human development on the number of incarcerated individuals. The extent which the roughness of HDI influences the roughness of the number of incarcerated individuals is given by

$$R^2 = 1 - \left(\lambda_{xy} - 1\right)_{(\lambda x \lambda y)1/2}$$

where:

$\lambda_{xy} = two-dimensional\ fractal\ dimension$
$\lambda_x = fractal\ dimension\ of\ x$
$\lambda_x = fractal\ dimension\ of\ x$

4.0 Results and Discussions

Figures 1 and 2 show the one-dimensional plots of HDI and number of incarcerated persons per thousand in various countries while Figure 3 shows the two-dimensional plot of HDI vs number of incarcerated persons.
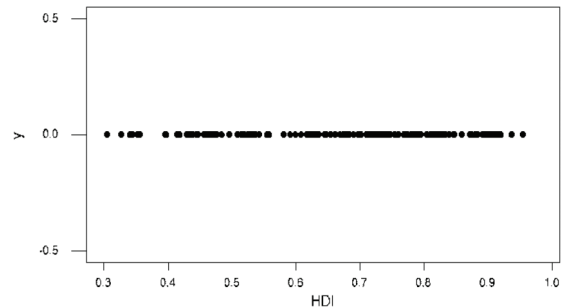

Figure 1. One-Dimensional Plot of Human Development Index of Countries
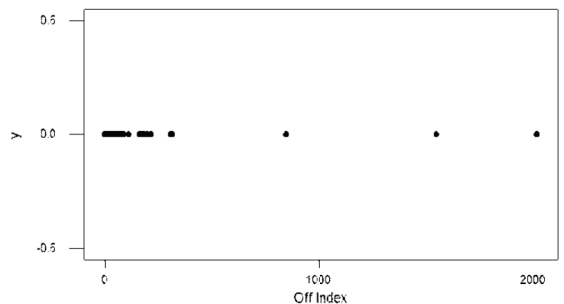

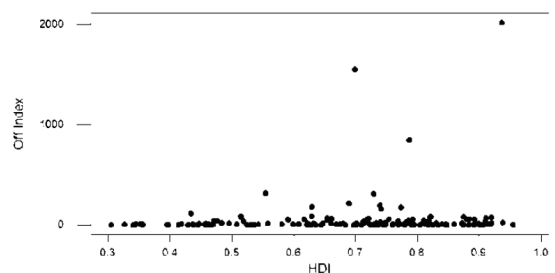**Figure 2. One-Dimensional Plot of Offenders' Index**

Figure 3. Two-Dimensional Plot of HDI vs Offenders' Index

Table 1 shows the summary of the fractal dimensions computed for the variables of the study.

Variable	Fractal Dimension
X: human Development Index	1.4985
Y: number of incarcerated persons	1.5795
XY: scatterplot	1.2814

Table 1. Summary of Fractal Dimensions

$R^2 = 0.908184$: fractal correlation

$R\lambda_{xy} = -0.028$ (p-value= 0.138) : correlation of fractal dimensions

$R_{xy} = 0.118$ (p-value=0.734) : correlation of original variables

DISCUSSIONS:

1. The one-dimensional plot reflects the variability of Human Development

Indices throughout the world. It is noted that the straight line has been fragmented into four distinct clusters representing low, medium, high and very high human development. The rest of the countries not belonging to the clusters are shown as isolated points on the one-dimensional plot. On the whole, the degree of fragmentation is calculated by its fractal dimension as 1.4985. The deviation statistic which describes the degree to which the data depart from a smooth straight line is roughly 49.85% or 50% deviation from smoothness. Countries belonging to the very high human development include 1st world countries from the West (with the exception of Asian countries like Singapore, Korea, Japan, Hongkong). The HDI rankings featured above were published in the 2013 Human Development Report: The Rise of the South: Human Progress in a Diverse World. On the other hand, the countries belonging to the low human development index include mostly African nations and some Asian countries like Yemen and Myanmar.

2. The One-Dimensional Plot of Offenders' Index is far more fragmented than the HDI with a fractal dimension of 1.5795 reflecting a smoothness deviation of close to 60% (57.95%). This means that the number of social offenders vary far greater than suggested by the variations in the HDI. This implies too many surprises which cannot be predicted using the traditional methods of analysis. In the case of United States of America, where HDI is reported to be the 3rd best amongst nations, its number of incarcerated offenders is high, in fact ranked number 1. (Human Development Report, 2013). However, in the case of Australia (to rank 2nd in terms of Human Development) reported a considerably smaller number of incarcerated offenders. In the case of Norway, ranking 1st in terms of HDI ranked 114 in number of incarcerated offenders. (The Eighth United Nations Survey on Crime Trends and the Operations of Criminal Justice Systems (2002): United Nations Office on Drugs and Crime, Centre for International Crime Prevention).

3. The two-dimensional plot reflects the impact of the variability of HDIs to the roughness of the number of incarcerated offenders. The degree of variability induced by the HDIs tends to decrease the variability of the number of incarcerated offenders as shown by the fractal dimension of the two-dimensional plot of 1.2814. Thus, the ruggedness or irregularities of the HDIs of various countries , on the whole, tended to pull down the inherent ruggedness in the reported

number of incarcerated offenders viz. high human development index induces a corresponding reduction in the number of offenders.

4. It follows that the key to minimizing the occurrence of deviant social behavior is the development of human potentials as reflected in the various dimensions of the human development index. We note in passing that this conclusions could not have been drawn if the ordinary correlation coefficient (Rxy = 0.118 (p-value=0.734)) were used because this Pearson measure would have indicated the opposite conclusion. In fact, the correlation of the fractal dimensions (R = -0.028 (p-value= 0.138) in conjunction with the fractal correlation coefficient (R2 = 0.908184) give a fairer picture of the actual magnitude of the relationship between the two variables.

5.0 Conclusion

The fractal correlation coefficient (fractogram) and the correlation of the fractal dimensions in tandem give a clearer picture of the degree of relationship induced by one highly variant variable on another variable than using the ordinary correlation coefficient. This is illustrated in the case of the relationship between the Human Development Index and the number of incarcerated offenders in various countries worldwide.

References:

Padua, R. N. & Borres M. S. (2013) "From Fractal Geometry to Fractal Statistics" (Recoleto Multidisciplinary Journal of Research, Vol. 1, No. 1, Research Journal of the University of San Jose-Recoletos, CHED-JAS Category B)

Padua, R. N. , Palompon, D. & Ontoy D. S. (2012) " Data Roughness and Fractal Statistics" (CNU Journal of Higher Education, Vol. 6, No. 1, CHED- JAS Category A, pp 87-101)

(The Eighth United Nations Survey on Crime Trends and the Operations of Criminal Justice Systems (2002) (United Nations Office on Drugs and Crime, Centre for International Crime Prevention)).

2013 Human Development Report: The Rise of the South: Human Progress in a Diverse World.

http://www.nationmaster.com/graph/cri_pri-crime-prisoners

www.epw.in/.../human-development-index-trends-1980-2012

Appendix

Rank	Countries	Offender's Index	HDI
# 1	United States	2019.234	0.937142
# 2	China	1549	0.699242
# 3	Russia	846.967	0.787681
# 4	India	313.635	0.554213
# 5	Brazil	308.304	0.72997
# 6	Thailand	213.815	0.689657
# 7	Ukraine	198.386	0.740395
# 8	South Africa	181.944	0.629413
# 9	Mexico	172.888	0.774959
# 10	Iran	163.526	0.741801
# 11	Rwanda	112	0.433863
# 12	Pakistan	87	0.514989
# 13	Indonesia	84.357	0.628666
# 14	Poland	80.467	0.82146
# 15	United Kingdom	78.753	0.875368
# 16	Germany	74.904	0.920098
# 17	Bangladesh	74.17	0.515418
# 18	Philippines	70.383	0.654358

# 19	Japan	69.502	0.911837	#57	Yemen	14	0.458294
# 20	Turkey	64.051	0.722224	#58	Syria	14	0.647709
# 21	Egypt	61.845	0.662291	# 59	Portugal	13.918	0.816447
# 22	Spain	59.251	0.884828	# 60	Dominican Republic	13.836	0.701998
# 23	Korea, South	58.564	#N/A	# 61	Zambia	13.173	0.447723
# 24	Kazakhstan	58.3	0.754233	# 62	Honduras	11.502	0.631604
# 25	France	56.957	0.893087	# 63	Lithuania	11.07	0.817518
# 26	Taiwan	56.225	#N/A	# 64	El Salvador	11.055	0.680417
# 27	Italy	55.67	0.880552	# 65	Israel	11.027	0.900217
# 28	Vietnam	55	0.617313	# 66	Ghana	10.992	0.558444
# 29	Morocco	54.351	0.590935	# 67	Moldova	10.903	0.660474
# 30	Colombia	54.034	0.719459	# 68	Panama	10.35	0.780434
# 31	Belarus	51.238	0.792867	# 69	Bulgaria	9.918	0.781833
# 32	Romania	48.075	0.785937	# 70	Libya	9.763	0.76898
# 33	Uzbekistan	48	0.654145	# 71	Greece	8.841	0.859577
# 34	Argentina	44.969	0.811317	# 72	Mozambique	8.812	0.326746
# 35	Tanzania	43.244	0.475956	# 73	Burundi	8.647	0.354809
# 36	Nigeria	40.447	0.471138	# 74	Belgium	8.605	0.896733
# 37	Malaysia	39.258	0.768805	# 75	Malawi	8.566	0.418269
# 38	Chile	36.636	0.819422	# 76	Costa Rica	8.526	0.772552
# 39	Canada	35.519	0.911155	# 77	Latvia	8.483	0.814019
# 40	Kenya	35.278	0.518915	# 78	Bolivia	8.315	0.674678
# 41	Algeria	34.243	0.712733	# 79	Guatemala	8.307	0.58116
# 42	Burma	32.797	#N/A	# 80	Ecuador	8.274	0.723953
# 43	Saudi Arabia	28.612	0.782481	# 81	Austria	8.114	0.895331
# 44	Peru	27.417	0.740929	# 82	Mongolia	7.871	0.67543
# 45	Tunisia	23.165	0.711817	# 83	Slovakia	7.758	0.840432
# 46	Australia	22.492	0.937985	# 84	Nicaragua	7.198	0.599227
# 47	Madagascar	20.109	0.483224	# 85	Nepal	7.132	0.463048
# 48	Cameroon	20	0.49547	# 86	Uruguay	7.1	0.791791
# 49	Sri Lanka	19.974	0.714919	# 87	Georgia	6.406	0.74455
# 50	Kyrgyzstan	19.5	0.622449	# 88	Cambodia	6.346	0.542692
# 51	Venezuela	19.255	0.748283	# 89	Angola	6.008	0.508236
# 52	Azerbaijan	19.136	0.733909	# 90	New Zealand	5.968	0.918843
# 53	Czech Republic	18.669	0.872564	# 91	Sweden	5.92	0.916179
# 54	Hungary	17.862	0.830843	# 92	Botswana	5.89	0.634418
# 55	Netherlands	16.93	0.921019	# 93	Jordan	5.589	0.700495
# 56	Singapore	16.31	0.894789				

# 94	Lebanon	5.535	0.744793		# 131	Congo	0.918	0.534255
# 95	Senegal	5.36	0.470034		# 132	Bahrain	0.911	0.795507
# 96	Switzerland	4.982	0.912922		# 133	Qatar	0.57	0.833585
# 97	Benin	4.961	0.436041		# 134	Gambia	0.45	0.438611
# 98	Namibia	4.814	0.608377		# 135	Djibouti	0.384	0.444859
# 99	Trinidad and Tobago	4.794	0.76049		# 136	Saint Lucia	0.365	0.724839
# 100	Jamaica	4.744	0.730494		# 137	Luxembourg	0.341	0.875384
# 101	Estonia	4.571	0.84601		# 138	Saint Vincent and the Grenadines	0.302	0.733134
# 102	Central African Republic	4.168	0.351861		# 139	Dominica	0.298	#N/A
# 103	Haiti	4.152	0.455996		# 140	Grenada	0.297	0.770466
# 104	Paraguay	4.088	0.668999		# 141	Malta	0.283	0.846882
# 105	Mali	4.04	0.344311		# 142	Antigua and Barbuda	0.186	0.759848
# 106	Chad	3.883	0.339568		# 143	Seychelles	0.157	0.805647
# 107	Denmark	3.435	0.901386		# 144	Saint Kitts and Nevis	0.135	0.745443
# 108	Finland	3.433	0.892478		# 145	Solomon Islands	0.134	0.529762
# 109	Papua New Guinea	3.302	0.465626		# 146	Sao Tome and Principe	0.13	0.524809
# 110	Swaziland	3.245	0.536007		# 147	Tonga	0.113	0.71045
# 111	Guinea	3.07	0.355177		# 148	Iceland	0.104	0.906476
# 112	Lesotho	3	0.461239		# 149	Palau	0.103	0.790562
# 113	Kuwait	2.946	0.790263		# 150	Vanuatu	0.093	0.626089
# 114	Norway	2.914	0.955202		# 151	Andorra	0.061	0.846316
# 115	Armenia	2.866	0.72877		# 152	Kiribati	0.055	0.628951
# 116	Burkina Faso	2.8	0.343389		# 153	Micronesia	0.039	0.64488
# 117	Croatia	2.611	0.804655		# 154	Marshall Islands	0.023	..
# 118	Mauritius	2.565	0.737115		# 155	Liechtenstein	0.018	0.883241
# 119	Togo	2.043	0.458929		# 156	Monaco	0.013	..
# 120	Suriname	1.933	0.683651		157	Nauru	0.006	..
# 121	Albania	1.532	0.748625		157	Tuvalu	0.006	..
# 122	Guyana	1.507	0.635504		159	Comoros	0	0.429406
# 123	Oman	1.403	0.730911		159	Tajikistan	0	0.621994
# 124	Mauritania	1.354	0.466979		159	United Arab Emirates	0	0.818085
# 125	Bahamas	1.28	0.793799		159	Uganda	0	0.456081
# 126	Cyprus	1.254	0.848007		159	Sudan	0	0.413597
# 127	Slovenia	1.099	0.891683		159	Cuba	0	0.779589
# 128	Belize	1.097	0.70169		159	Niger	0	0.304114
# 129	Barbados	0.992	0.825183					
# 130	Fiji	0.982	0.701727					

159	Turkmenistan	0	0.697511
159	Ethiopia	0	0.395667
159	Zimbabwe	0	0.397179