# Assessment of Students' Learning on a Fractal Viewpoint

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## *Abstract*

*This study explores the fractal dimensions of students' cognitive skills according to the levels of difficulty using fractal model approach and analysis. The data utilized were from the test results based on the competency-based constructed items. Findings revealed that data sets obey a non- normal distribution as depicted in histograms and normality tests that yield to fractal statistical analysis. Hence, subjects with lesser fractal dimension and disparity value tend to be less rough and rugged whereas subjects with higher fractal dimension and high disparity value is perceived to be more irregular. This result has a great impact in exploring the fractality of test scores that will lead to a deeper understanding on students' authentic performance with a direct, relevant and realistic application of learning as emphasized in the implementation of Outcomes-based Education (OBE).*


*Keywords: assessment, fractal dimensions, fractal analysis, test scores, students' learning*

## 1.0 Introduction

Assessment is a vital part and an important issue in the teaching and learning cycle of education. It requires proper planning and appropriate tools prepared by teachers. Assessment, however, is defined as an ongoing process- continuous undertaking of the teacher usually done before, during and after instruction (Angelo, 1995). Educational assessment as perceived by majority of students is a difficult subject most especially when it comes to test constructions in both thelow and high level thinking skills and validating toolsof learning. All have the same dilemma including the time constraints in answering, the level of difficulty, poor scores, understanding the mathematical problems and worst, guessing of responses through situational analysis.The experiences of many students in the process of assessment are still based on behaviorist approach where essential facts and skills are measured and assessed, where grading and ranking are the primary goals in evaluation (Niss, 1993).

Educational assessment as a context of educational measurement and evaluation is a method of evaluating personality in which an individual solves variety of lifelike problems. According to Cronbach, as cited by Jaeger (1997), there are three principal features identified in assessment: (1) the use of a various techniques; (2) correlating observations in structured and unstructured situations; and (3) combination and application of information. Bloom (1970) describes the educational assessment as a process which characteristically starts with an analysis of the criterion and the environment. It is clearly evident that educational assessment focuses not only on

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what is to be learned, how to learn it but also to the nature of the learners. With this fact, the strengths and weaknesses of learners can be identified, at the same time the effectiveness of instructional materials used in the curriculum. If assessment is solely for the purposes of attaining grades, ranks and credentials, assessment practices reflect the diversity found in the learners themselves (Swan, 1993). The recognition of students' diversity showed a great shift in the vision of assessment toward a system based on evidence, outcomes-based assessment and authentic assessment as the new trends in assessing students' learning. This shift is toward relying on the professional judgment of teachers and away from using only externally derived evidence (NCTM, 1995, p. 2). Testing procedures may be an assessment of learning (formative and diagnostic) in nature, assessment on learning(summative) in nature and assessment as learning (self and peer) in nature (Clarke, 1988; Mitchell & Koshy, 1993; NCSM, 1996; Stenmark, 1991).Learning is fractal as describes by Stephen in his article" Toward Real Educational Testing". It is not temporary but permanently affects the learning process. Through this truth, using grades to measure learning uniformly to diverged learners is a spurious endeavor (McGreggor, et.al, 2010). Similarly, the quantification of test measurement for an individual's IQ, or for a person's personality or for human being's honesty are all equally spurious tests. Hence, all learning and tests for learning require consciously meaningful example of this teaching. Educational set-up must create authentic tests for learning to knowingly resemble academic experiences for any truth of learning. In effect, learning acquired from visual and sensual experiences through fractal testing in schools have genuinely positive significance to life.

Through this study, researchers will be able to predict the ruggedness, self-similarities, and scale in variance of test scores in Assessment. This will identify which subject is difficult and easy. Thus, the findings of the study contribute to the depth understanding and the nature of the subject, strategies in teaching, methods and efforts of learning. Furthermore, it explains the domino effect of assessment of teaching in smaller scales such as classroom-based instruction to district, then regional, national and even international phenomenon and issued related to the findings.

**Conceptual Framework**

This part presents the relationship of educational assessment to fractals. It explains the ruggedness and irregularities of performance in assessing students' learning. Fractal Analogy to the Raven's Test of Intelligence explains the fluctuations of intelligence scores of an individual. This study explains that human's intelligence is fractal. Thus, results of test scores show great irregularities. Assessing students' learning in the world of formal education seeks to define academic excellence and a high quality performance. This is the primary goal of the existence: to survive the challenges of life and the changing world through learning.

On this matter of assessment in education, the researcher believed that the performance of students in any subject or focus of learning is not normally distributed. To analyze the students' performance, the researcher utilized the test results of both the assessment of students' learning (ASL) 1 and 2 of the school year 2012-2013. This data provide comparison on the level of difficulty of the given subjects. Test is constructed based on the specified competencies set as the standards of learning as designed by the Commission on Higher Education.

**Table 1. Test Scores Data**

| Subjects | Number of Students | Number of Items | Highest Score | Lowest Score |
|---|---|---|---|---|
| **ASL 1** | **138** | 140 | 135 | 28 |
| **ASL 2** | **125** | 90 | 63 | 15 |

As presented in Table 1 above, Assessment of Student Learning (ASL) 1 is an education subject that focuses on the different theories, principles and guidelines in test construction, types of educational assessment, and testing validity and reliability of tests through item analysis. ASL 2 is an authentic assessment or alternative assessment that stresses on the application of the different theories and principles in writing and assessing different types of test using the performance assessment. This is to develop actual skills in educational assessment using rubrics and any assessment tools.

Classical assessment and evaluation theory define item difficulty index as the proportion of students obtaining the correct answer for that item to the total number of responses for that item:

1….P = R/T , where

R is the number of correct responses and T is the total number of responses (i.e., correct + incorrect + blank responses).

Hence, the higher this index value, the lower is the difficulty, and the greater the difficulty of an item, the lower is its index. This index is counter-intuitive since it actually measures the "item easiness" rather than its difficulty. For the purposes of this study, we propose to define "item difficulty" in the following manner:

2…. $Q_j = 1 - P_j, \quad j = 1,2,\ldots,T$

where T is the number of items, is now a monotone function of the difficulty level. The test difficulty, as a whole, is defined as the average of the item difficulties:

3…. Test Q = $\frac{\sum_1^T Q_j}{T}$

**Table 2. Traditional Test Characteristic**

| Subjects | No. of Students | No. of Items | Test Difficulty |
|---|---|---|---|
| ASL 1 | 138 | 140 | 0.69 |
| ASL 2 | 125 | 90 | 0.42 |

**Moreover, Table 2, gives the traditional test characteristic using the formulas indicated above. It shows that ASL 1 is more difficult than ASL 2.**

## Research Methodology

The data were collated and tabulated for fractal analysis in testing the normality of the data sets, histogram, probability plot, scatter plot, time series, and computation of probability of density functions using the equation of fractal inferential statistics presented by Padua, et.al. (2013). Mathematically, a monofractal distribution is described by the power-law probability density:

$$(1)\quad f(x;\lambda) = \left(\frac{x}{\theta}\right)^{-\lambda},\ \lambda > 1,\ x \geq \Theta$$

It is shown in Padua, et.al (2013) that the maximum likelihood estimators of λ and Θ are respectively.

$$(2)\quad \lambda = 1 + n\left(\sum_{i=1}^{n} \log\left(\frac{x_1}{\theta}\right)\right)^{-1}$$

$$(3)\quad \Theta = min\,\{x_1, x_2, \ldots, x_n\}.$$

A practical approach suggested in estimating λ is to plot log f(x) versus log() and to use the slope of the line as estimators of λ. This could be heuristically argued by taking the logarithm of both sides of (1):

$$(4)\quad log\,f(x) = log\,(\ ) - \lambda\,log\left(\frac{x}{\theta}\right).$$

Moreover, the indicators of monofractality as presented by Padua, et.al (2013) fit to a fractal distribution f(x) to the quantile of the distribution G(.).

Let $(x_{(a)})$ be the $\alpha^{th}$ quantile of G(.):

$$(5)\ldots\ G\,(x_{(a)}) = \alpha$$

At each of αth quantile of G(.), we fit a power law distribution F(.) such that:

$$(6)\ldots\ G\,(x_{(a)}) = F(x_{(a)}) = \alpha,$$

or equivalently, obtain:

$$(7)\ldots\lambda_{(\alpha)} = 1 - \frac{\log(1-\alpha)}{\log\left(\frac{x_\alpha}{\theta}\right)},\ \text{for all } \alpha \in (0,1).$$

Denote the empirical quantiles by X(αk) where αk = , 1≤ k≤ n-1. An estimate of λ can be obtained from (7):

$$(8)\ldots\ \lambda = \frac{1}{n-1}\sum_{k=1}^{n-1} \lambda_{(\alpha k)}.$$

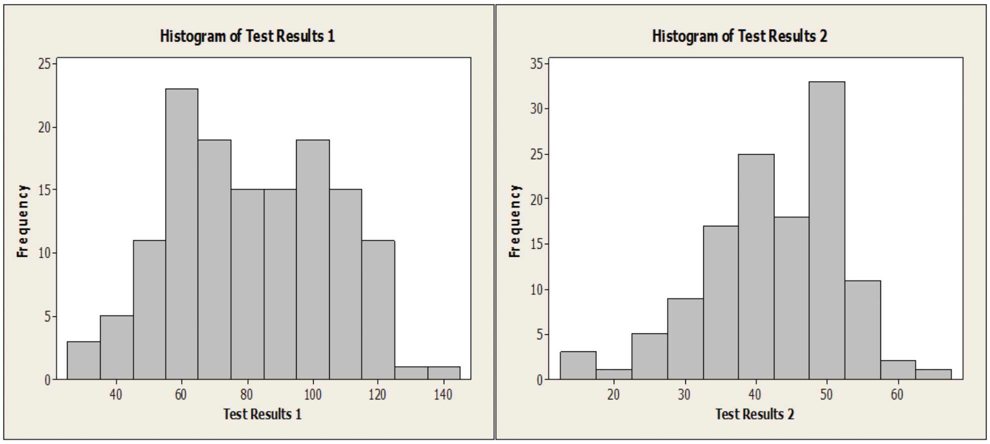



**Figure 1. Histogram of Assessment of Student Learning 1 and 2**

The histogram of the test performance of ASL 1 and 2 students shows a non-normal distribution as presented in Figure 1

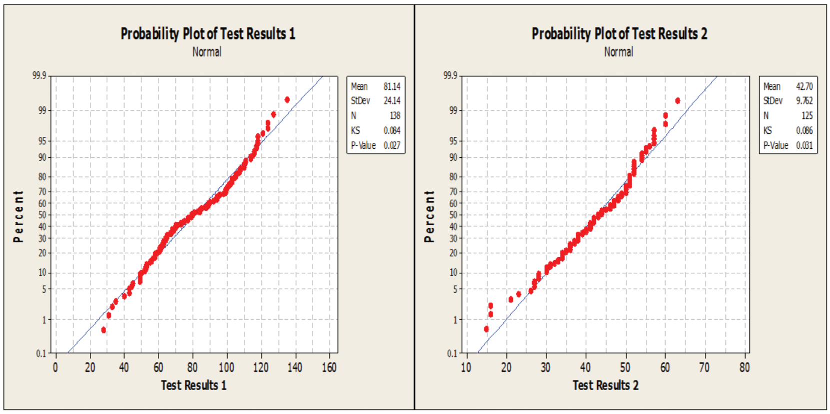



**Figure 2. Probability Plot of Assessment of Student Learning 1 and 2**

Based on the figure 2 above, the density of probability of data sets, with p-values of 0.027 and 0.031 respectively are lesser than 0.05. It depicts the non-normality of scores using the Normality test.

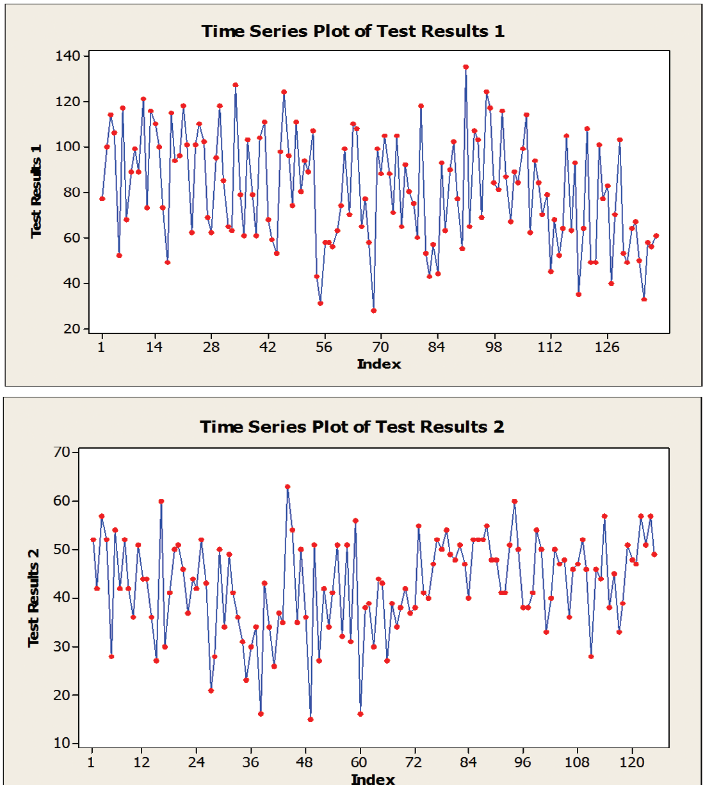



**Figure 3. Time Series Plot of Assessment of Student Learning 1 and 2**

The figure 3 displays the data as time series of values of a non-normal distribution of test scores. It has been noticed in the graphs that the set of scores show irregularities, gradual fluctuations and ruggedness of students' scores along with time.

## Fractal Model and Analysis of Educational Assessments

After thorough computations of the estimated values of lambda from the test scores, the following figures show a definite pattern of exponentially distributed random variates.

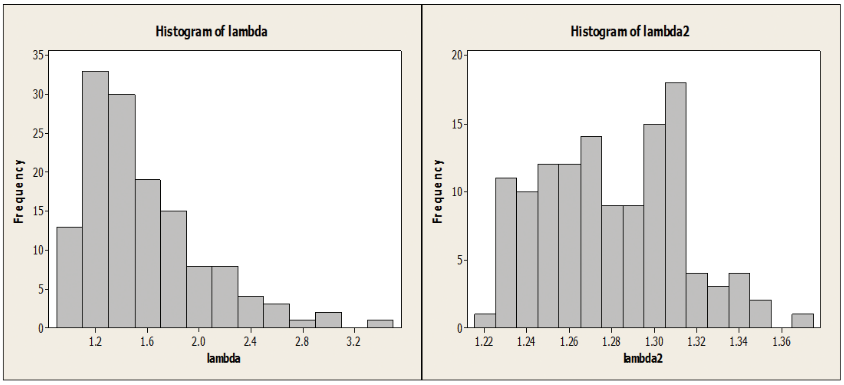



**Figure 4. Histogram and Probability Plot of Lambda 1 and 2**

From the non-normal distribution of the computed lambda as shown in figure 4 the density function can be written as:

(9)…$g(\lambda) = Ae^{-k\lambda} = ke^{-k(\lambda-1)}$ $\lambda > 1$

It indicates that the histogram and the probability plot of lambda follows a pattern of irregularities.

Computing the estimated lambdas (λ) is the preliminary step in determining the fractal spectrum. Table 3 below shows the descriptive statistics pertinent to the fractal dimensions of the two educational subjects being compared.

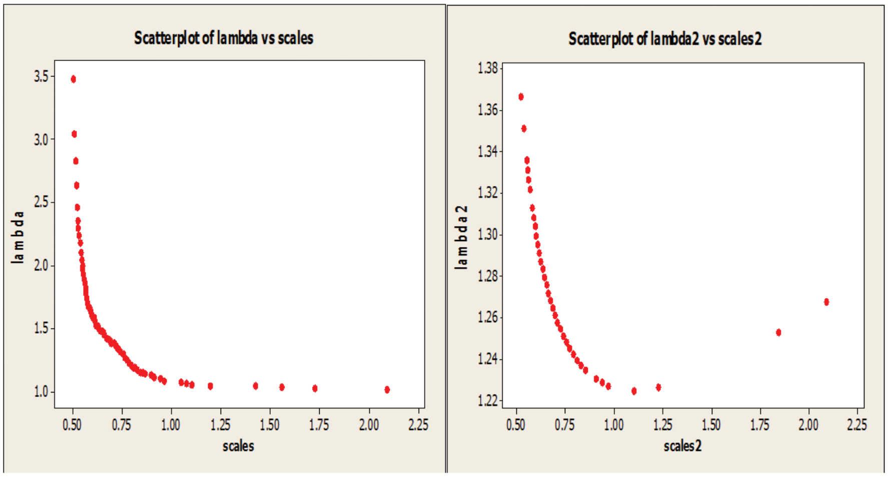
**Table 3. Descriptive Statistics of Fractal Dimensions**

| Variables | N | Mean | SE Mean | StDev | Minimum | Median |
|---|---|---|---|---|---|---|
| lambda 1 | 137 | 1.5738 | 0.0401 | 0.4689 | 1.0152 | 1.4479 |
| lambda 2 | 125 | 1.2797 | 0.00288 | 0.0322 | 1.2246 | 1.2753 |

It has been noted on the Table 2 above that ASL 1 has higher average fractal dimension of 1.5738 than ASL 2 with 1.2797. Additionally, comparing the average fractal dimensions of the said subjects cannot suffice a claim or a conclusion in terms of its ruggedness and difficulty. Barabat, et al (2013) proposes that to explicitly show the fractality of data sets, the scale invariance must be established using a scale (S) written as:

$$S = \frac{1}{\log\left(\frac{x}{\theta}\right)}$$

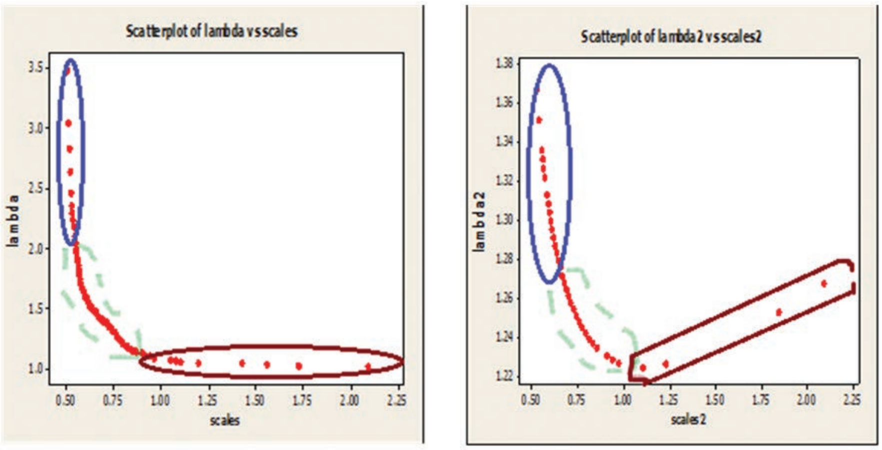
Using the computed fractal dimensions and the scale measures of each subject, the fractal spectrum were plotted as shown below:




(a) (b)

**Figure 5. Estimated Fractal Spectrum of Test Scores**

It has been noted in Figure 5 that the fractal spectrum of lambdas versus its scales represents the fractality of the two subjects. It is clearly evident that the graphs are distinctly different. Also, lambdas concentrate in smaller scales. This means that high scores decompose in lower scales that create intense ruggedness. As seen in Figure 5b, low scores are dispersed and were scattered on larger scales. In this sense, to show the disparity of fractals, the difference between low and high scores in the subject as a change in its slope in the graph were scaled and computed. Figure 6 below represents the fractal disparity of the test scores.




(a) (b)

**Figure 6. Fractal Dimensions Movement of Test Scores**

With reference to Figure 6 (a and b), the researcher divided the data sets according to the shift and movement of its slope in the graph into small, medium and large scales. The average lambda and test scores were identified from the small and large scales data and raised to the power of the average lambda in the said scales. The difference between the two values manifests the disparity between the two scales. Hence, it leads to a claim of the level of difficulty of the subjects. This process is sum-up within the table below (see table 4 and 5T).

| ASL 1 | Average of data (a) | Average of lambda (b) | a^b |
|---|---|---|---|
| Small Scale | 106.74 | 1.41 | 724.34 |
| Large Scale | 41.38 | 1.13 | 67.14 |
| Disparity | | | 657.20 |

**Table 4. Fractal Disparity between Scales of ASL 1**

| ASL 2 | Average of data (a) | Average of lambda (b) | a^b |
|---|---|---|---|
| Small Scale | 52.28 | 1.32 | 185.44 |
| Large Scale | 35.67 | 1.25 | 87.17 |
| Disparity | | | 98.27 |

**Table 5. Fractal Disparity between Scales of ASL 2**

## Relating Test Difficulty and Fractal Dimensions

After establishing the fractal dimensions and its difficulty index, researchers proceeded to find the significant relationship of the two measures. This is to show the consistency of its findings. The results are summarized in figures 7 and 8 and supported by tables 6 and 7.

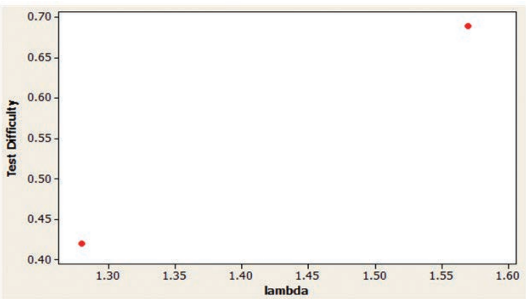



**Figure 7: Graph of Test Difficulty versus Fractal Dimension**

**Table 6: Relationship Between Test Difficulty and Fractal Dimension**

The regression equation is
Test Diff = 0.182 + 0.254 lambda

| Predictor | Coef | SE Coef | T | P |
|---|---|---|---|---|
| Constant | 0.18194 | 0.01819 | 48.49 | 0.000 |
| lambda | 0.254461 | 0.009520 | 26.73 | 0.000 |

S = 0.0667416 R-Sq = 84.2% R-Sq(adj) = 84.1%

Tables 4 and 5 above show the disparity of the subjects' fractal dimensions. It is greatly seen that ASL 1 has higher disparity value than ASL 2. Accordantly, with the outcome of comparison in table 2 and estimated fractal spectrum in figure 5, it can be inferred that ASL 1 is more difficult than ASL 2. Moreover, there appeared to be a perfect matching between subjects found difficult (subjects with high difficulty indices) and subjects with high fractal dimensions.A significant linear relationship exists between fractal dimensions and difficulty indices. Empirical model obtained states:Test Diff = 0.182 + 0.254 lambda with an R-squared value of 84%. The relationship indicates that higher fractal dimensions imply higher test difficulty(see Figure 6). Hence, a subject with high fractality implies a more difficult subject (see figure 5a) while low fractality and ruggedness (see figure 5b) is interpreted as less difficult subject. Moreover, a teacher has to be guided on the varied strategies in teaching by means of thorough efforts on the basic mathematical operations in education and fundamental theories and principles of learning. Students have hard time grasping all the necessary skills, testing techniques and mathematical computations which in later part of learning becomes the application in ASL 2.

## Conclusion

Using fractal analysis, the researcher concluded that the subject with higher fractal dimension (λ= 1.5738)and with higher disparity value(see Table 3)is more rugged, more irregular, and rough than a subject with lesser fractal dimension (λ=1.2797) and lesser disparity value(see Table 4). Students' performance congested in smaller scales only and a few in both the medium and larger scales. This means that ASL 1 which is the fundamental subject in assessment of student learning is more difficult than ASL 2. In similar term, basic and fundamental subjects are more difficult to comprehend than the authentic assessment as a true application of the theories and principles of its basic elements in learning. It implies that: (1) the methods of teaching fundamental and basic education must not be apart from the embedded content and nature of the subject. (2) The fundamentals of learning must be given with higher focus and intensive emphasis to grasp the significant skills and mastered those prerequisite abilities and principle to apply it to the real-life situations. (3) Strategies of instruction have to be well planned and well implemented to impart appropriate mind treatment of diverse learners. (4) Basic education as a whole has the same level of impact and importance to the secondary education and even to the tertiary level of education, so trainings of these levels must be on skill acquisition and mastery of necessary skills for the next level of learning.

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